Spectral analysis of galaxies

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A living set of notes of a graduate course in the making... A little-theory-but-many-hands-on-exercises course.

Preamble: These notes grew out of the need to give my students a hands-on experience similar to the one I (and older folks like me) had to develop by myself before the *there's-a-python/IDL/etc-package-somewhere-which-does-it-all* fashion caught in. Useful as these tools may be, there's just no substitute for DIY insofar as learning something is concerned. These notes are all about DIY, the Jurassic way. The emphasis is really on basic DIY spectral analysis of galaxies, not on using the latest/fanciest methods/models/data. Some of the exercises are actually downright silly, while others are probably useless in serious work, but still useful to develop basic skills, and some I have not tried myself before. Also, sometimes the student is lead to do something just to realize, later on, that (s)he's done a mistake, like applying a recipe/formulae where it does not apply. Finaly, there are very few references or detailed explanations of anything. The teacher is supposed to provide them while teaching. Also, there are no plots, as they'll all be done by the students themselves! Programming skils are required. Students should be able to handle ascii files and tables, perform numerical integrations and other simple operations, as well as plot results.

1 SSP spectra from BC03

Get http://minerva.ufsc.br/∼cid/PG2012/BC03models.tar.bz2. This tar file contains BC03 spectra of SSPs os 221 different ages and 6 different metallicities (m22 ... m72, or m122 ... m172), 2 different IMFs (chab & salp) and two sets of evolutionary tracks (Padova1994 & Padova2000). Each of these $221 \times 6 \times 2 \times 2 = 5304$ *-spec files is a simple 2 columns ascii table with λ (in Å) and the luminosity spectrum produced by an SSP whose initial total mass is 1 $M_\odot$, with λ running (unevenly sampled) from 91 to 1600000 Å. The units of l_λ are $L_\odot$/Å$M_\odot$. The tar file further contains 4 tables

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Base.bc03.Padova1994.chab.All
Base.bc03.Padova1994.salp.All
Base.bc03.Padova2000.chab.All
Base.bc03.Padova2000.salp.All
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to help you find the age (t) and metallicity (Z) of each *.spec file for a given choice of IMF and tracks.

All of this comes from the evolutionary synthesis models of BC03, a seminal work in the field. Despite known problems (some of which have been fixed, others not), it is still the main reference in the field, and an awfully useful place to start our work.

You should get familiar with these spectra! This is the goal of the exercises below. BC03 recomend the Chabrier + Padova1994 models (in Base.bc03.Padova1994.chab.All), so look at these first, but write your scripts general enough to change to some other set of models easily.

Exercise 1.1 - *The overall goal here is that you get a feeling of how SSP spectra change as a function of t and Z, both in amplitude, global shape (say, colors), and detailed shape (absorption lines). Write an illustrated essay on what you found. The following are general tips and stuff that you should include in your essay.*

1. *Make a script/macro which makes a movie (sequence of plots) showing the sequence of $l_\lambda(t, Z, \text{IMF}, \text{tracks})$ spectra from the 1st to the last age in the models, for fixed Z, IMF and tracks. Make it flexible enough to allow you to change the x and y scales.*

2. *To illustrate what Z does to an SSP spectrum, make plots of $l_\lambda(t, Z)$ for a fixed t and all different Z's. Chose $\log t/yr = 6$, 6.5, 7.0, $\ldots$ 9.5, 10, 10.3. Do this for each of the 4 sets of IMF/tracks.*

3. *To illustrate what the IMF does to an SSP spectrum, compute the ratio of $r = l_\lambda(\text{chab})/l_\lambda(\text{salp})$ and plot it as a functin of λ. Do this for $\log t/yr = 6$, 6.5, 7.0, $\ldots$ 9.5, 10, 10.3, and one Z at a time. Then pick $\lambda = 5635$ and plot r as a function of t. Do this separately for Padova1994 & Padova2000 tracks.*

4. *To illustrate what the choice of evolutionary tracks does to an SSP spectrum, overplot the l_λ's obtained from Padova1994 and Padova2000. In the same figure, show a panel with the ratio of the two spectra. Do this for $\log t/yr = 6$, 6.5, 7.0, $\ldots$ 9.5, 10, 10.3, and one Z at a time. Use the salp IMF throughout.*

Be verborragic on figure captions. In fact, you don't need a long text. Just describe briefly how you deal with the data and leave discussions/descriptions for fig. captions.

Exercise 1.2 - Rectified/high-pass spectra. *Differences in amplitude and continuum shape make it difficult to compare variations in the absorption line strengths and profiles for different t's and Z's.*

One way to circumvent this is to rectify the spectra, dividing it by a fucntion which represents its broad-range ("low-pass", in the lingo of fourier filtering of signals) behaviour.[1]

We'll do this folowing the recipe of Mathis et al (2006; MNRAS, 365, 385). They use a box-car filter of length $\delta = 500$ Å *to compute, for each* λ, *the average of* l_λ *in the* $l_\lambda \pm \delta/2$ *window:*

$$l_\lambda^{LP} = \frac{1}{\delta} \int_{\lambda-\delta/2}^{\lambda+\delta/2} l_\lambda(\lambda - \lambda') d\lambda' \tag{1}$$

One then divides the original l_λ *by this low-pass, smoothed version of it, obtaining an adimensional "high-pass" spectrum:*

$$s_\lambda^{HP} = \frac{l_\lambda}{l_\lambda^{LP}} \tag{2}$$

which should be a flat $s_\lambda^{HP} \sim 1$ *function except in regions containing absorption lines.*

Let's use this trick to inspect the BC03/chab/Padova1994 SSPs in the $\lambda = 3650$ *to 8500* Å *range. For every* t *and* Z, *compute the corresponding* $s_\lambda^{HP}(t, Z)$ *in this range. (Note that you'll need the* l_λ*'s in the* $3650 - \delta/2 = 3500$ Å *to* $8500 + \delta/2 = 8750$ Å *range to compute the low-pass spectrum* l_λ^{LP}*.)*

♣ *Overplot* $s_\lambda^{HP}(t, Z)$ *for a fixed* Z *and different* t*'s. Tip: You may need to split* t *into ranges and pick a few values to obtain a clear plot. For instance, one figure may be for* $\log t = 8$, *8.2, 8.4, 8.6, 8.8 and 9.0, and another one for* $\log t$ *from 9 to 10.2.*

♣ *Overplot* $s_\lambda^{HP}(t, Z)$ *for a fixed* t *and the 6 different* Z*'s.*

Besides helping you visualize things, rectified spectra are sometimes used to analyse the data. The codes STECKMAP and ULySS, for instance, adopt this strategy, and MOPED can also be used in this way. STARLIGHT has done so once (in Coelho et al 2009), but with an inefficient version. Removing the global shape (ie, rectifying) is necessary when your flux calibration is not reliable on long λ scales (say, you have a blue/red calibration problem because of badly corrected atmospheric extinction, or you had clouds or problems while observing the standard stars). The downside is that rectification throws away the continuum shape, which, as you must have realized by now, does contain lots of information on the stellar populations.

Exercise 1.3 - *Another way to study spectral variations as a function of* t *and* Z *is through derivatives like*

[1] Note: This procedure is sometimes called "normalization" instead of "rectification". I use "normalization" to mean dividing a spectrum by a single number, not a function. Beware!

$$D^t_\lambda(t,Z) \equiv \left(\frac{\partial \log l_\lambda}{\partial \log t}\right)_Z$$

$$D^Z_\lambda(t,Z) \equiv \left(\frac{\partial \log l_\lambda}{\partial \log Z}\right)_t$$

In principle, D^t_λ and D^Z_λ should show you which spectral regions are more sensitive to t and Z. These need not (and will not) be the same for all t's and Z's, of course, so that, for instance, some λ's will be more sensitive to t for t around 10^8 yr while other regions will have higher derivatives for t of order 10^{10} Gyr. You'll have to do it and see.

♣ *Explore these ideas and produce pretty plots. I can think of a nice 3D-image with λ in the x-axis, t in the vertical one and a color coded intensity map representing either $D^t_\lambda(t,Z)$ of $D^Z_\lambda(t,Z)$. The interpretation may be tricky, but in general terms you should see here what you've found earlier looking the spectra (or their high-pass version) directly.*

♣ *You may want to try replacing l_λ by s^{HP}_λ in the derivatives to get a cleaner picture of absorption lines strengths variations with t and Z.*

Note: Practical evaluation of these functions may require some computational tricks, since we have discrete tables, while derivatives are meant to be applied to continuous functions. Use your imagination (or some python package) to deal with this. Ah: I've never done this myself! In fact, I suspect it's hard to make it work properly...

This is a long, laborious and rather tedious series of exercises. Because so much of what we'll do relies on SSP models, it's important you do your best here.

2 Building composite stellar populations

Galaxies are not single SSPs. Some may be reasonably approximated in this way (like massive ellipticals), but most aren't, particularly spirals and irregulars, where the existence of multiple generations is evident to the eye in any color image.

It is formaly simple to compute the theoretical spectrum corresponding to a system with an arbitrary star formation history (SFH). All you need is to specify a function $\psi(t)$ such that $\psi(t)dt = dM(t)$ gives you the mass formed in stars between t and $t+dt$. This mass dM in stars which nowadays have age t will contribute a luminosity $dL_\lambda = l_\lambda(t)dM(t) = l_\lambda(t)\psi(t)dt$ to the galaxy spectrum. Adding up all generations leads to

$$L_\lambda = \int_0^T l_\lambda(t,Z)\psi(t)dt \tag{3}$$

where T is a generic upper limit ($T \leq 14$ Gyr, the age of the Universe). A full description of the SFH would require a description of the Z distribution of stars formed at each t, which we will not do here. Assuming that all the gas which is forming stars at a given t has the same Z, chemical evolution can be plugged in eq. 3 by specifying a $Z = Z(t)$ function in the argument of $l_\lambda(t, Z)$. This is an interesting exercise, but for our purposes here it will be sufficient to keep Z constant.

Piece of cake. But what function should I chose for $\psi(t)$? Once one choses a shape for $\psi(t)$, limits can be put on its amplitude such that it does not over/under predict the L_λ observed for the source(s) you want to match. Still, there are infinite possibilities for the shape of $\psi(t)$.

A widely popular choice is a decaying exponential starting at $t = t_0$:

$$\psi(t) = \begin{cases} Ae^{-(t_0-t)/\tau} & \text{if } t \leq t_0; \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

where τ controls the effective lenght of the SF period. $\tau \to 0$ leads you to an SSP-like regime, while for $\tau \to \infty$ one has a constant SFR regime. The birthdate t_0 defines the age of the oldest stars seen nowadays. The amplitude A defines the peak SFR, as well as the total stellar mass formed since $t = t_0$.

Exercise 2.1 - *For the exponentially decaying SFR of eq. 4, show that*

$$A = A(M_\star, t_0, \tau) = \frac{M_\star}{\tau} \frac{1}{1 - e^{-t_0/\tau}}$$

where $M_\star$ is the total mass converted into stars during the galaxy life.

♣ *At which age t_X is $X\%$ of $M_\star$ already formed?*

♣ *For $t_0 = 10$ Gyr and $\tau = 5$ Gyr, compute t_X for $X = 20$, 50, 80 and 90%. Mark these ages and X fractions in a plot of formed mass as a function of t.*

Exercise 2.2 - *Write a code which computes the predicted L_λ spectrum for an exponentially decaying SFR. Use the BC03 $Z_\odot$ SSP spectra for a Chabrier IMF and Padova 1994 tracks.*

♣ *Plot $L_\lambda(t = 0 = \text{now})$ for $t_0 = 0.1$, 1 and 10 Gyr, and $\tau/t_0 = 0.1$, 1 and 10. Use $M_\star = 1M_\odot$ for all cases. The amplitudes of the resulting spectra will be very different. It's instructive to make two plots of L_λ: One in absolute units ($L_\odot/\text{Å}M_\odot$) and the other in units of the L_λ at, say, $\lambda = 5635$ Å.*

♣ *For each of these $3 \times 3 = 9$ models, compute the mass $M_\star$ necessary to match an observed luminosity of $4.0 \times 10^6 L_\odot/\text{Å}$ at $\lambda = 5635$ Å.*

♣ *For the spectrum of the $t_0 = 1$ Gyr and $\tau = 10$ Gyr model, plot (in a single frame): (a) the total L_λ; (b) the L_λ associated to stars formed from $t = 1$ to 0.5 Gyr, and (c) the L_λ produced by the stars formed in the last 0.5 Gyr. What do you conclude from the plot?*

ATT! As you'll hopefuly find out for yourself in the exercise above, there are numerical issues to solve when your SFR function varies substantially on time scales comparable to the age sampling of the $l_\lambda(t, Z)$ SSP spectra... The conversion of 3 to a summation needs care. Interpolating $l_\lambda(t, Z)$ to a fine t-grid is one possible solution. Another one is to compute, analitically, the mass $\Delta M_i = \int_{t_i^{low}}^{t_i^{upp}} \psi(t)dt$ in each of the t_i-bins in the SSP age-grid ($i = 1 \dots 221$), and replace eq. 3 by

$$L_\lambda = \int_0^T l_\lambda(t, Z)\psi(t)dt \approx \sum_i l_\lambda(t_i, Z)\Delta M_i \tag{5}$$

You'll have to figure out which option is best. Sanity checks include comparing the spectrum for some $\tau \ll t_0$ (i.e., a very small θ) with that of an SSP of age t_0. They should be very similar, in shape and amplitude.

Exercise 2.3 - *For a generic $\psi(t)$ function, derive analytical expressions for:*

1. $\langle t \rangle_M$ = *the mass-weighted mean stellar age*
2. $\langle \log t \rangle_M$ = *the mass-weighted mean stellar log age*
3. $\langle t \rangle_L(\lambda)$ = *the mean stellar age weighted by its contribution to the light at λ*
4. $\langle \log t \rangle_L(\lambda)$ = *the mean stellar log age weighted by its contribution to the light at λ*

♣ *Compute $\langle t \rangle_M$ and $\langle \log t \rangle_M$ = for the 9 (t_0, τ) combinations of the previous exercise.*

♣ *Compute $\langle t \rangle_L(\lambda)$ and $\langle \log t \rangle_L(\lambda)$ for the same 9 models, and plot the results as a function of λ. Yes, these are ages which depend on wavelength!*

It has recently become popular to invert the sign of the exponent, such that SF rises from some $t_1 > t_0$, peaks at t_0 and then shutts off. Other variants allow for randomly located bursts, defined by, say, box-car functions $\psi_{burst}(t; t_b, \tau_b, M_b)$ which form a mass M_b in the period from $t = t_b$ to $t_b - \tau_b$. Such bursts are then added to a more smoothly varying ψ. No one has yet proposed a sinusoidal ψ, but hey, why not? The sky is the limit! How are you going to decide which SFH represents your data best? Surely, given so much liberty, many possibilities will yield similar results (ie, same or very similar predicted observables), and you'll never know the real one. Degeneracy is part of this game.

A popular (and powerful, but not perfect) approach to this issue is to bypass the hopeless quest for "the" unique solution and admitt precisely the opposit, ie, that everything is possible! It's just that some solutions are more likely than others. The general idea is to build a huge library of models for wildly different SFHs. For each model is this collection you tabulate a series of observables (spectral indices, colors, or the full spectrum L_λ itself). You then compare each to your observations, and compute its likelyhood through, say, its χ^2—for gaussian errors, the likelyhood is $\propto e^{-\chi^2/2}$. Now you have a long table with one likelyhood for each model SFH. Then you start making questions like: What is the stellar mass $M_\star$? What is the mass-weighted mean age of the stars? And etc. The answer will not be a number, but a probability distribution function (PDF), where all models (no matter how crazy) are included (crazy models will have low likelyhood, and thus weight little on your PDF). PDFs may be cumbersome to work with but have many advantages, one of which is that it allows you evaluate uncertainties. Maaaaanny papers use this approach nowadays. The early papers by Brinchmann, Gallazzi and Kauffmann on SDSS data, are examples. We'll do this ourselves in this course soon.

3 SFR indicators

The previous chapters we purely theoretical. The ones below, starting here, are more observationally geared. We'll make use of the models above, but now thinking of ways to relate them to the real world. This section focuses on ways of estimating the recent SFR of galaxies. It's still purely theoretical in the sense that no observational data are used, but at least we're getting closer to real data.

3.1 Using SSPs-spectra to devise/test possible SFR indicators

Everyone estimates SFRs somehow. There are several subtleties and implicit assumptions in this game, so let's try to do these ourselves from scratch.

Let

- $dM(t) = \psi(t)dt$ be the mass turned into stars between t and $t + dt$.
- $l(t)$ be a function which describes the evolution of some generic radiative output *per unit formed mass*[2] of an SSP. l can be the luminosity (per unit formed mass) at some wavelength λ (in which case its natural units are $L_\odot/\text{Å}M_\odot$). It can also represent the luminosity in a given filter

[2] When talking about masses of stellar systems one must always distinguish between the mass which was turned into stars from the one which remains in stars. This is because stellar evolution ejects stuff back into the ISM though winds, SNe, PNe, etc., so that not all that go into stars stays in stars—in fact, you and this pdf file are made of excreted stellar material! As much as 50% of the mass of an SSP can go back to the ISM during a Hubble time, depending (among other things) on the IMF. This is why "formed mass", "initial stellar mass" and other awkward expressions appear in these notes. Keep this in mind.

(units of $L_\odot/M_\odot$), or the number of ionizing photons produced per unit time and mass (units photons/$sM_\odot$).

The goal is to compute the l-thing for a composite stellar population resulting from a period of *constant* SFR from T years ago till today ($t = 0$). Hopefully, this will lead to an empirical (though model dependent) way of measuring the SFR (ψ) out of some observable related to l.

The amount of l-light we receive from stars formed t years ago is just

$$d\Lambda(t) = l(t)dM(t)$$

so that adding all the stars formed since $t = T$ (just like in eq. 3) we would see, today, a total of

$$\Lambda = \Lambda(T, \psi) = \int_0^T l(t)dM(t) = \psi \int_0^T l(t)dt \tag{6}$$

where we have already assumed that $\psi(t) =$ constant in the last T years, and zero before that.

Clearly, if $l(t)$ is something which decays quickly with time (like the ionizing radiation, which is mainly produced by massive, short-lived stars), old populations would add up little or nothing to the integral, so the integral should converge quickly. Conversely, if l is, say, the luminosity at a red wavelength, where stars of all masses/ages contribute, the integral should increase steadily with T. Things which increase indefinitely with T (like the latter example) will not give you a good estimator of the recent SFR.

We'll now play with different choices for l, computing and ploting Λ versus T. This should give us clues as to which observables are useful to estimate SFR's.

Exercise 3.1 - *Use the BC03 model SSP spectra for a Chabrier IMF and Padova 1994 tracks, to compute (for each Z separately),*

1. *The monochromatic luminosities per unit formed mass (call them l_λ) at $\lambda =$ 1528, 2271, 3543, 4020, 4770, 5635, 6231, 6533, 6593, 7625, 9134 Å, and 1.2, 1.6, 2.2,3.4, 4.6, 12 and 22 μm. (No, these are not random choices!)*
2. *The H-ionizing photon rate per unit formed mass (q_H). To do so, integrate what has to be integrated (...) for energies ≥ 13.6 eV ($\lambda \leq 912$ Å).*
3. *The bolometric luminosity per unit initial mass, ie, $l_{bol} = \int l_\lambda d\lambda$.*

♣ *Plot the l_λ's, q_H and l_{bol} against the age t, one curve for each of the 6 metallicities (m22, m32 ...m72). Judging by the looks of your plots, evaluate (qualitatively) which ones are more suitable to* *estimate the recent SFR.*

Exercise 3.2 - *Apply eq. 6 to each of the different incarnations of $l(t)$ in the exercise above. Present the results as plots of the corresponding Λ/ψ curves as a fucntion of T. Use T-values equal to the 221 ages in the BC03 files.*

Again, juding by the looks of your plots, evaluate which ones are more suitable to estimate the recent SFR. This time, you can be more quantitative, and estimate, for each tracer, the time-scale over which it provides a useful estimator of ψ. One way of doing this is to compute the value of T for which Λ reaches, say, 90% of its asymptotic value (if there is one!).

Exercise 3.3 - *The Hα luminosity is a champion SFR indicator (e.g., Kennicutt 1998, ARAA). From recombination theory, we know that one out of each 2.22 ionizing photons produces an Hα photon. Luckly, this number changes very little as a function of nebular conditions like temperature and density.*

Use your numbers for q_H and the results for the corresponding Λ to estimate the coefficient k in

$$\frac{SFR}{M_\odot yr^{-1}} = k \times \frac{L(H\alpha)}{L_\odot}$$

Note 1: There will be one k for each Z. Compare your coefficients with those in the literature.

Note 2: Implicit assumptions here include that no ionizing radiation escapes the nebula, that $L(H\alpha)$ has been corrected for extinction, and that dust does not eat much of the $h\nu > 13.6$ eV photons (at least it does not compete with HI in this sense).

Exercise 3.4 - *Re-do all the above using BC03 models for a Salpeter IMF. Interpret the differences wrt the Chabrier IMF.*

Exercise 3.5 - *Re-do all the above using BC03 models for Padova 2000 tracks.*

Exercise 3.6 - *NGC 604 is an HII region in the nearby galaxy M33. It's ionizing star cluster is about 3 Myr old, and the (extinction corrected) Hα luminosity is $10^{39.63}$ erg/s. Estimate its SFR. (Since NGC 604 has a gas-phase metallicity well below solar, I suggest you use the k coefficient for the m42-models/$Z = 0.2 Z_{\odot}$.)*

♣ *Now think about what you've done ... Was it a wise thing to apply your $SFR = kL_{H\alpha}$ relation to this object? (One-word-tip: age!)*

♣ *If estimating the SFR for NGC 604 was not a good idea, what else could you derive from knowledge of its Hα luminosity and age? (Tip: Given an answer in solar masses.)*

3.2 Dust emission as a SFR-tracer.

Dust eats light and reprocesses it into thermal emission in the far infra-red. Modeling the dust emission is a complex issue, much beyond my powers, but all we need here is a simple energy-budget kind of calculation.

For an uniform dust-screen in front of a source which emitts a L_λ^0 spectrum, the transmitted (ie, observed) luminosity is $L_\lambda = L_\lambda^0 e^{-\tau_\lambda}$. The total energy eaten up by dust and reprocessed into FIR radiation is thus

$$R = \int_0^\infty (L_\lambda^0 - L_\lambda) d\lambda = \int_0^\infty L_\lambda^0 \left(1 - e^{-\tau_\lambda}\right) d\lambda \tag{7}$$

which can be equalled to the total FIR luminosity (L_{FIR}), an observable quantity (with caveats related to how the FIR range is sampled by actual observations).

Note that R depends on the quantity (τ_V) of dust and its "quality" (the $q_\lambda \equiv \tau_\lambda / \tau_V$ extinction law). It is convenient to factor scales out expressing R in units of the intrinsic bolometric luminosity of the system, $L_{bol} = \int_o^\infty L_\lambda^0 d\lambda$, leading to the reprocessed fraction

$$r = r(\tau_V; q_\lambda) \equiv \frac{R(\tau_V; q_\lambda)}{L_{bol}} = \frac{\int_o^\infty L_\lambda^0 \left(1 - e^{-\tau_\lambda}\right) d\lambda}{\int_o^\infty L_\lambda^0 d\lambda} = \frac{L_{FIR}}{L_{bol}} \tag{8}$$

From previous work, you should have $l_{bol}(t, Z)$, the bolometric luminosity per units mass of an SSP of age t and metallicity Z, i.e., the denominator of r. Before going into SFR-related stuff, it's usefull to play with r for SSPs.

Exercise 3.7 - *Forget IMF, tracks and Z for a while, and use only BC03 SSPs for the Chabrier IMF and $Z_{\odot}$ Padova 1994 tracks. Assume a simple $q_\lambda = (\lambda/5500\text{Å})^{-0.7}$ law and compute/plot $r(t)$ for SSPs* *of different ages (all 221 of them) for $\tau_V = 0$, 1, 2, 3, 5, 10.*

Exercise 3.8 - *Extinction laws (q_λ) are known from the UV to the optical and beyond, but little is known about its behaviour below the Lyman limit, where neutral H and dust particles compete to see who is going to eat an ionizing photon first. In other words, we know little about q_λ for $\lambda \leq 912$ Å. To bracket possible solutions, take the two limits: $q_\lambda = 0$ and $q_\lambda = \infty$ for $\lambda \leq 912$ Å. Do this and compare the resulting $r(t)$ plots with those obtained above.*

Exercise 3.9 - *Repeat the exercise above for q_λ following a Calzetti law. You'll have to find it from the literature!*

At this stage you should be able to obtain an expression for R for the case of a constant SFR between $t = 0$ (now) and T. As before, the resulting expression should depend on ψ and T (from eq. 6), but now τ_V and your choice of q_λ should also appear in the math. The result is

$$R = R(T; \tau_V, q_\lambda) = \psi \int_0^T \int_o^\infty l_\lambda^0 \left(1 - e^{-\tau_\lambda}\right) d\lambda \, dt \tag{9}$$

As usual, to scale-away things it may be useful/convenient to adimensionalize things, expressing R in units of the corresponding L_{bol}

$$L_{bol} R(T) = \psi \int_0^T \int_o^\infty l_\lambda^0 d\lambda \, dt \tag{10}$$

(though this is not necessary for the exercises below).

Exercise 3.10 - *Plot $R(T; \tau_V, q_\lambda)/\psi$ curves as a fucntion of T. Use T-values equal to the 221 ages in the BC03 files, the Calzetti law, and overplot curves for $\tau_V = 0$, 1, 2, 3, 5, 10.*

♣ *What do you conclude about the dependence on T? Is $R = L_{FIR}$ a useful indicator of the SFR (ψ)? Over which time scales?*

♣ *If you were to write*

$$\frac{SFR}{M_\odot yr^{-1}} = k \times \frac{L_{FIR}}{L_\odot}$$

what value of k would you take, and what would be the undelying assumptions in your derivation/estimation of the SFR?

♣ *Compare your k coefficients with those in the literature.*

♣ *Many people interpret L_{FIR} as originating from dust heated by the same young stars responsible for powering emission lines like Hα and many others. Based on your experiments, do you agree with this?*

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4 Magnitudes

Why use a constant minus 2.5 times the log of the thing instead of the thing itself? Magnitudes suck, but you've gotta live with them. We'll soon analyse some magnitude based data on real galaxies, so let's open a parenthesis to get used to this system.

Suppose you measure some kind of "flux" Q (say the photon flux, or an energy flux, maybe monochrmoatic or maybe in a band through a filter, counts in a CCD, etc). You'd express it as magnitudes through

$$m = -2.5 \log Q + ZP = -2.5 \log \frac{Q}{Q^{ZP}} \tag{11}$$

where $ZP \equiv 2.5 \log Q^{ZP}$ is a zero-point, such that a source with a flux Q^{ZP} has $m = 0$ (by definition). There are different systems to define Q's and their ZP's. The most popular nowadays seems to be the AB system, which is defined in spectroscopic terms. Suppose you have a source whose flux density spectrum is f_ν, in $\mathrm{ergs\,s\,cm^{-2}\,Hz^{-1}}$. The corresponding AB magnitude at frequency ν is defined as

$$m_{AB}(\nu) = -2.5 \log f_\nu - 48.60 \tag{12}$$

such that a source with flat spectrum $f_\nu = f^{ZP} = 3.631 \times 10^{-20}\ \mathrm{ergs\,s\,cm^{-2}\,Hz^{-1}}$ will have an AB magnitude = 0 for any ν.

Ok, but so far this is just an awkward way of rewriting f_ν. Let's now think of the magnitude in a broadband filter (say, the r filter) coupled to a photon counting device such as a CCD. When you do an observation of a source whose energy spectrum is f_ν, you will collect Q photons

$$Q = A \times t \times \int \frac{f_\nu}{h\nu} R_\nu d\nu \tag{13}$$

where A is the telescope area, t is the exposure time and $\frac{f_\nu}{h\nu}$ is the photon flux[3] in units of photons $\mathrm{s}^{-1}\,\mathrm{cm}^{-2}\,\mathrm{Hz}^{-1}$. R_ν is the response function of the system (atmosphere, telescope, detector, filter transmisivity, ... the whole thing). It tells you that for every N photons coming from the source you will only detect $N \times R_\nu$ of them. For our purposes, pretend R_ν represents the filter transmissivity profile, but it in facts embeds everything else.

If you were to point to a $f_\nu = f^{ZP} = 3.631 \times 10^{-20}$ ergs s $\mathrm{cm}^{-2}\,\mathrm{Hz}^{-1}$ source you would count

$$Q^{ZP} = A \times t \times \int \frac{f^{ZP}}{h\nu} R_\nu d\nu \tag{14}$$

photons. The AB magnitude of your source, through your chosen R_ν filter, is thus

$$m_{AB} = -2.5 \log \frac{Q}{Q^{ZP}} = -2.5 \log \frac{A \times t \times \int (f_\nu/h\nu) R_\nu d\nu}{A \times t \times \int (f^{ZP}/h\nu) R_\nu d\nu} \tag{15}$$

$$m_{AB} = -2.5 \log \frac{\int f_\nu \nu^{-1} R_\nu d\nu}{\int \nu^{-1} R_\nu d\nu} - 48.60 \tag{16}$$

where the -48.60 comes from $+2.5 \log f^{ZP}$. (Sometimes one writes $d\nu/\nu$ as $d \ln \nu$, or even as $d \log \nu$, since the $\log e$ factors would cancel out anyway).

Notice that what you have inside the log is a weighted mean value of f_ν, where the weight is R_ν/ν.

$$\overline{f_\nu} = \frac{\int f_\nu (R_\nu/\nu) d\nu}{\int (R_\nu/\nu) d\nu} = 10^{-0.4(m_{AB}+48.60)} \tag{17}$$

and notice too that the assumption that we are using a photon counting device is impregnated in this definition. If our detector measures energy (say, a bolometer instead of a CCD), then the weight is just R_ν and the ν^{-1} factors disappear.

In our case, we'll want to compare models and data, so we'll need to convert model fluxes to AB magnitudes, or, conversely, transform observed AB magnitudes to (weighted mean) fluxes. From the last equation you see that this is not so hard: All we need to know is R_ν.

Exercise 4.1 - From $\nu \to \lambda$. *We've used f_ν's so far, but in the UV-optical-IR its is customary to work with f_λ's, in units of ergs $\mathrm{s}^{-1}\,\mathrm{cm}^{-2}\,\text{Å}^{-1}$. The SSP spectra we played with, for instance, are given in per Å form, not in per Hz.*

All we need to know to convert things is that $f_\nu d\nu = f_\lambda d\lambda$ and $\nu\lambda = c$. Show that

$$m_{AB} = -2.5 \log \frac{\int f_\lambda \lambda R_\lambda d\lambda}{\int \lambda^{-1} R_\lambda d\lambda} - 2.41 = -2.5 \log \frac{\int \lambda^2 f_\lambda (R_\lambda/\lambda) d\lambda}{\int (R_\lambda/\lambda) d\lambda} - 2.41 \tag{18}$$

[3] **Important note:** Notice that $h\nu$ appears because we are counting photons. If, instead, our detector counts energy, the $h\nu$ would not appear in this equation.

♣ *Just to make sure you're on the ball, which units for f_λ and λ are assumed?*

Note that going from $\nu \to \lambda$ screws up the notion that the thing which goes inside the log is a weighted mean flux density, ie, we cannot associate the argument of the log to some $\overline{f_\lambda}$ as we did in eq. 17 for $\overline{f_\nu}$. In this λ-based formulation, the argument of the log is, instead, a weighted mean of the product $\lambda^2 f_\lambda$, with weight $= R_\lambda/\lambda$.

$$\overline{\lambda^2 f_\lambda} = \frac{\int \lambda^2 f_\lambda (R_\lambda/\lambda) d\lambda}{\int (R_\lambda/\lambda) d\lambda} \tag{19}$$

This unfortunate fact of life comes about because a flat f_ν spectrum, which is entrained in the definition of AB magnitudes, is not flat in f_λ; in fact, f_ν = constant implies that $\lambda^2 f_\lambda = c f_\nu$ = constant, so that $f_\lambda \propto \lambda^{-2}$. To reinforce what was just said, strange as it may seem, m_{AB} gives you a $\overline{f_\nu}$, but not a $\overline{f_\lambda}$!

The reason I say unfortunate is that we are all used to f_λ, but to work with AB mags we need to switch to a slightly different thing, namely, $\lambda^2 f_\lambda$. There's a relatively clean way of sorting this mess, but before showing it, lets first illustrate that working with f_ν's is no such big deal afteral.

Exercise 4.2 - Translating BC03-SSP spectra to absolute AB mags. *The bc2003*.spec SSP files you've downloaded give you l_λ in units of $L_\odot$Å$M_\odot$. If your SSP has an (initial) stellar mass $M_\star$, its luminosity density spectrum is $L_\lambda = M_\star \times l_\lambda$. Put it at a (luminosity) distance d away and its flux will be*

$$f_\lambda = \frac{L_\lambda}{4\pi d^2} = \frac{M_\star l_\lambda}{4\pi d^2}$$

or, written in terms of f_ν

$$f_\nu = \frac{L_\nu}{4\pi d^2} = \frac{M_\star l_\nu}{4\pi d^2} = \frac{M_\star l_\lambda \lambda^2}{4\pi d^2 c}$$

You'll now compute m_{AB} for a distance of 10 pc, such that the apparent magnitude equals the absolute one, M_{AB}. Throughout, keep $M_\star = 1 M_\odot$, so that your fluxes and magnitudes reflect that of an idealized $1 M_\odot$ SSP at $d = 10$ pc. You'll need to know that $L_\odot = 3.826 \times 10^{33}$ erg/s, and that 1 pc $= 3.086 \times 10^{18}$ cm.

Before doing actual filter-photometry, let's do "monochromatic photometry", ie., compute M_{AB} for a single frequency (wavelength). Use the BC03 Chabrier/Padova1994 models. Our M_{AB}'s will depend on λ, t and Z, and also on $M_\star$.

(a) *Plot $M_{AB}(\lambda)$ vs. λ for SSPs of 10^6, 10^7, 10^8, 10^9 and 10^{10} yr and the 6 metallicities in BC03. Make the plots on the same scale to facilitate comparisons.*

(b) *Plot $M_{AB}(\lambda)$ vs. $\log t$ for the following λ's: 3543, 4770, 6231, 7625 and 9134 Å. Do this only for $Z = Z_\odot$ (the m62 models). Not coincidentaly, these are the central wavelengths of ugriz SDSS filters.*

(c) *What is the $M_{AB}(\lambda = 6231)$ of a $10^{11} M_\odot$, 10 Gyr, $Z_\odot$ SSP? What would be its m_{AB} for a distance of 100 Mpc?*

(d) *Let's now do filter photometry. Re-do exercise b above, this time using the actual response functions R_λ of SDSS ugriz filters. (For filter profiles, see our course web deposit).*

(e) *Compare the "monochromatic" M_{AB}'s with those just derived. As long as a filter is not too wide and/or does not cover regions of wild spectral variations (spectral breaks), the results should be similar.*

Exercise 4.3 - *If you hate all this mess and insist on working with f_λ's—as I do but perhaps you shouldn't!—you can always redefine the weight and obtain a different kind of weighted mean f_λ which does correspond neatly to a measured AB magnitude.*

♣ *For a weight $r_\lambda \equiv R_\lambda \times \lambda$, show that*

$$\overline{f_\lambda}(m_{AB}) = \frac{\int f_\lambda r_\lambda d\lambda}{\int r_\lambda d\lambda} = \frac{1}{\lambda_\star^2} 10^{-0.4(m_{AB}+2.41)} \tag{20}$$

where $\lambda_\star$ is a funny kind of weighted mean wavelength representing the filter, defined by

$$\frac{1}{\lambda_\star^2} \equiv \overline{\lambda^{-2}} = \frac{\int \lambda^{-2} r_\lambda d\lambda}{\int r_\lambda d\lambda} \tag{21}$$

Conversely, you may want to express m_{AB} in terms of these new definitions

$$m_{AB} = -2.5 \log \lambda_\star^2 \overline{f_\lambda} - 2.41 \tag{22}$$

This trick allows one to carry on doing averages in λ space. Since I'm used to seeing spectra in f_λ, I can use this to plot a f_λ spectrum and overplot a point corresponding to a photometrically measured $\overline{f_\lambda}$.

♣ *Compute $\lambda_\star$-values for the ugriz SDSS filters.*

OBS: These are things I just invented to satisfy my own philosophical needs. Not that it's a relevant "discovery", but I just want to note that I did not find it in the literature (including google!), though *it's surely there somewhere.*

One needs experience to make sense of a statement like "the r-band magnitude of the source is $m_r = 17.7$ in the AB system", or to get a feeling of how many suns would fit in a $M_{AB}(r) = -21.0$ galaxy. (Magnitudes suck, did I say that before?) Yet, when some one says "the photometric uncertainty is ± 0.13 magnitudes" (in whichever system), one immedeately knows that the fluxes are measured with a $\sim 13\%$ accuracy. This is because, as you can easily show, an error Δm relates to an error in $\overline{f_\lambda}$ through

$$\Delta m = (2.5 \log e) \frac{\Delta \overline{f_\lambda}}{\overline{f_\lambda}} \sim \frac{\Delta \overline{f_\lambda}}{\overline{f_\lambda}} \tag{23}$$

I've used $\overline{f_\lambda}$ here, but the same applies to $\overline{f_\nu}$.

5 DIY SED fitting

SDSS	M_{FUV}^{obs}	M_{NUV}^{obs}	M_u^{obs}	M_g^{obs}	M_r^{obs}	M_i^{obs}	M_z^{obs}	M_J^{obs}	M_H^{obs}	M_K^{obs}
B	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
G	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
R	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$

Table 1: Absolute AB mags for 3 galaxies (K-corrected and derredened by Galactic extinction).

SED = Spectral Energy Distribution. This may mean different things for different people. Technically, it can mean a proper spectrum, f_λ, measured λ-by-λ. But most (including me, I and myself) take SED to mean a sparse set of flux measurements, like ugriz photometry of a system.

The goal here is to combine what we learned about stellar population models (§1 and §2) and how to deal with AB magnitudes (§4) to use photometric data on galaxies to infer something about their properties (masses, SFHs, etc.). This is the first time you'll start applying your just acquired skills to actual data! We'll play with magnitudes but exactly the same formalism/procedure can be used for other observables.

This whole section will be about using (some or all of) the data in Table 1 to infer/estimate properties like the stellar mass, mean age, mean Z, extinction, etc. The table gives AB magnitudes from FUV to K for 3 galaxies, named B, G and R (for obvious reasons). Since everything you do will have to be done 3 times, write your programs smartly enough!

The general script of this section goes like this:

1. Build a reference library of models.

2. Find the best model fit for galaxies B, G and R.
3. Apply simple bayesianities to produce a revised/refined analysis of the same data.
4. Re-do things for different subsets of the data in Table 1.

As usual in these notes, you'll do all of the work yourself. To get started, let's use only a subset of these data (the ugriz photometry) and build our tools and skills from there.

5.1 ugriz magnitudes: Building the tools

Consider only the absolute ugriz magnitudes of B, G & R in Table 1. We want to us these data to infer/estimate stellar population properties, ie, to do a "from data to parameters" journey.

You'll first need a library models to compare your data with. Let's keep things as simple as possible and treat galaxies as SSPs, such that their stars formed all at the same time and have the same metallicity. A bad assumption, surely, used for two purposes: Simplify things, and illustrate that bad assumptions do not prevent you from working! (Afteral, you can always fit a parabola with a straight line and estimate it's a and b coefficients ...) You'll later see that it's trivial to expand/change your library to incorporate more realistic SFHs. We will, however, introduce one minor complication: Extinction. Let's get to work.

Exercise 5.1 - Building a reference library. *Once again, let's stick to the BC03/Chabrier/Padova1994 model set. There are 1326 SSP spectra there (221 t's $\times$ 6 Z's). Let's make 21 versions of each of these, by applying a Cardelli, Clayton & Mathis (1989) reddening-law, with $A_V = 0, 0.1 \dots 1.9, 2.0$. In case you forgot, $l_\lambda(t, Z, A_V) = l_\lambda(t, Z, A_V = 0) \times 10^{-0.4 A_V q_\lambda}$, where $q_\lambda = A_\lambda / A_V$. (If you're too lazy to type in all the coefficients in the CCM law, the file ExtLaws.out lists q_λ^{CCM} in its 2^{nd} column.) Note: The CCM law does not span the same λ-range as the BC03 spectra, but that's Ok, since we only require reddened spectra in the range of our filters. In other words, apply $\times 10^{-0.4 A_V q_\lambda}$ in the 1000—33333 Å range for which the CCM law is defined and pretend $q_\lambda = 0$ elsewhere.*

For each of the $1326 \times 21 = 27846$ models, compute the ugriz M_{AB}'s for a $1 M_\odot$ SSP. Since these M_{AB}'s are for a SSP borned with just 1 $M_\odot$, lets call them $M_b^{mod}(t, Z, A_V, 1 M_\odot)$, where b denotes the band. Organize your results in a table with 27846 lines and $3 + 5 = 8$ columns: In the first 3 columns store the parameters t, Z and A_V. In the following 5 columns store the predicted ugriz M_b^{mod}'s.

Exercise 5.2 - Finding the best fit. *Now that we've built our reference library, let's find which of these 27846 models best matches each of the B, G and R galaxies in Table 1. By "best match" we'll adopt the conventional least-χ^2 (maximum-likelyhood) criterion. We thus seek to minimize*

$$\chi^2 = \chi^2(t, Z, A_V, M_\star | M_b^{obs}, \sigma_b) = \sum_b \left(\frac{M_b^{obs} - M_b^{mod}(t, Z, A_V, M_\star)}{\sigma(M_b^{obs})} \right)^2 \quad (24)$$

Just browse through your M_b *(where* $b =$ *u, g r, i and z) library-table and you'll quickly realize that none of your models comes even close to matching the* M_b^{obs}*'s of B, G nor R. That's because our library was built for a* $M_\star = 1M_\odot$ *"mini-galaxy", while the* M_b^{mod} *which goes in the above equation is that for a generic mass* $M_\star$*. We need to scale up our model* M_b*'s to realistic galaxy stellar masses. A stupid way of doing it is to add yet another dimension to our library grid, computing* M_b*'s for, say,* $\log M_\star/M_\odot = 8.0, 8.1, 8, 2 \dots 12.9, 13.0$*, boosting* $N_{library}$ *from 27846 to 1420146. You do not wanna do that!*

Here's how you get around this problem. $M_\star$ *has a trivial effect on our models: It just scales things up linearly in luminosity. It should be clear that for a generic* $M_\star$ *the absolute magnitude would be*

$$M_b^{mod}(t, Z, A_V, M_\star) = M_b^{mod}(t, Z, A_V, 1M_\odot) - 2.5 \log M_\star/M_\odot$$

Hence, $M_\star$ *just adds a constant term to the* M_b*-values in our library. Now ask yourself this: For a given choice of* (t, Z, A_V)*, what is the value of* $M_\star$ *which best matches the* M_b^{obs} *data? The answer comes from solving*

$$\frac{\partial \chi^2}{\partial M_\star} = 0 \quad (25)$$

which you shoud DIY to find that the optimal $M_\star$ *satisfies*

$$2.5 \log M_\star(t, Z, A_V | M_b^{obs}, \sigma_b) = \frac{\sum w_b^2 \left[M_b^{mod}(t, Z, A_V, 1M_\odot) - M_b^{obs} \right]}{\sum w_b^2} \quad (26)$$

where we've defined $w_b \equiv 1/\sigma_b$*. This result is really trivial. The RHS is just the (weighted) mean difference in magnitudes between our* $1M_\odot$ *SSP and the source. The LHS tells you the mass which minimizes this difference. You now know, for each of your* (t, Z, A_V) *27846 models, what* $M_\star$ *to use in order to best match the ugriz data for galaxy B, G or R.*

♣ *Do the calculations. Organize your results in a table with 27846 lines and* $4+1+5 = 10$ *columns. In the first 4 store* t*,* Z*,* A_V *and* $M_\star$*. In the* 5^{th} *column store the* χ^2 *of this model, and in the remaining ones store the predicted* M_b^{mod} *values for* $b =$ *u, g, r, i and z. There should be one such table for each of B, G and R.*

♣ *Pick the best model (smallest* χ^2*) you find and fill in Table 2.*

♣ *For didatic purposes, also fill in Table 3, where* $\delta_b = (M_b^{obs} - M_b^{mod})/\sigma_b$ *measures the (adimensional) distance between models and observatons in units of the corresponding uncertainty (ie, each of the*

SDSS	t [yr]	$Z/Z_\odot$	A_V	$M_\star/M_\odot$	χ^2	M_u^{mod}	M_g^{mod}	M_r^{mod}	M_i^{mod}	M_z^{mod}
B										
G										
R										

Table 2: Best fit models for galaxies B, G & R.

SDSS	δ_u	δ_g	δ_r	δ_i	δ_z	$\overline{\Delta}$ [mag]
B						
G						
R						

Table 3: Residuals from best fits.

terms in the χ^2 summation). Decent fits should have $\delta_b \sim 1$. The last column in this alternative table is an alternative figure of meritt, defined as

$$\overline{\Delta} = \frac{1}{N_b} \sum_b |M_b^{obs} - M_b^{mod}| \tag{27}$$

which gives you a feeling of how good your fit is. This is not a conventional statistic (like χ^2), but it's easily interpreted and does not depend on the (sometimes hard to compute) errors σ_b.

♣ *Now that you have found a best model, you may want to plot its λ-by-λ $M_{AB}(\lambda)$ spectrum, overploting the 5 photometric points following the tips and formulae in §4. If all went well, the model should be close to the data points!*

5.2 Bayesianities and a probabilistic/statistical re-analysis of the ugriz data

Well done. You've fitted the data. But, are you sure your best fit is a unique solution? Clearly not. Other models in your own library surely give χ^2's nearly as good as the best you found, but for $(t, Z, A_V, M_\star)$ parameters which are not necessarely close to the ones you got. Some sort of statitical analysis/error estimation is in order. As mentioned in §2, the way to do this is to build PDF's. The "P" in PDF means probability, and probabilities are the realm of Bayesian statistics.

Each of the $i = 1 \ldots 27846$ models in your library has an associated likelyhood $\mathcal{L}_i$, which measures the probability of observing the data you got given an assumed set of parameters and the errors. Assuming that the b =u, g, r, i and z measurements are independent (not correlated), it follows that the probability of measuring M_u^{obs} and M_g^{obs} and $\ldots M_z^{obs}$ is the product of the individual $P(M_b^{obs}|i)$ probabilities for each filter. Further assuming gaussian errors for each M_b^{obs}, one has

$$\mathcal{L}_i \propto e^{-\frac{1}{2}\delta^2_{u,i}} \times e^{-\frac{1}{2}\delta^2_{g,i}} \ldots \times e^{-\frac{1}{2}\delta^2_{z,i}} = e^{-\frac{1}{2}\chi^2_i} \tag{28}$$

where $\delta_b = (M_b^{obs} - M_b^{mod})/\sigma_b$ are the same dimensionless residuals used in Table 3.

To assign a probability to model i you should also specify a *prior* expressing things you know about the parameters *before* looking at the data! Sounds odd but you do this all the time, even when you do not realize it! In our case, priors were already built in our library. One of them states that $0 \leq A_V \leq 2$, with equal probability for any value in this range, i.e., $P(A_V)$ = constant for $0 \leq A_V \leq 2$, and $P(A_V) = 0$ otherwise. The inclusion of SSPs-only is also an implicit prior assumption (a pretty bad one, by the way). You always have priors, since you always make assumptions!

Denoting the set of parameters by a vector p_i and denoting the data by D (including all M_b^{obs} and σ_b values), the *posterior* probability of model i given the data is

$$P(p_i|D) \propto \text{ prior } \times \text{ likelyhood } = P(p_i)P(D|p_i) \tag{29}$$

The difference between prior and posterior is, of course, what you have learned from the experiment. If $P(p_i|D)$ is to behave like a probability, it should add up to 1, so:

$$P(p_i|D) = \frac{P(p_i)P(D|p_i)}{\sum_i P(p_i)P(D|p_i)} = \frac{P(p_i)e^{-\chi_i^2/2}}{\sum_i P(p_i)e^{-\chi_i^2/2}} \tag{30}$$

Notice that you have all you need to compute the RHS of this equation.

Since the index i represents a given set of parameters, ie, $p_i = \vec{p_i} = (t_i, Z_i, A_{V,i}, M_{\star,i})$, our $P(p_i|D)$ gives you a multidimensional joint probability. Nice, but often hard to work with. It's often more useful/convenient to project some of the dimensions of the parameter-space to construct PDFs for one or two parameters. This is called marginalization. Let's see how to do this in practice.

Exercise 5.3 - Bayesian SED analysis - estimating 1D PDFs. *Let's focus on a single parameter, t, and ignore all the others. The way our library was built we already have a pre-established grid of t values (the 221 ages in BC03), but let's pretend we do not just to make things more general. Define yourself a grid of ages, say, uniformly spaced in its log: $x_k = x_0 + k \times \Delta x = 0 \ldots N_x - 1$, where $x \equiv \log t/yr$. Bin x_k corresponds to $\log t$ in the $x_k \pm \Delta x/2$ range. Chosing $x_0 = 5$, $\Delta x = 0.1$ and $N_x = 53$ is enough to cover the whole $\log t$ range with good resolution, but you should play with Δx to see if and how it affects your results.*

In the previous exercise you've built a table with t, Z, A_V, $M_\star$ and the corresponding χ^2, one table for galaxy B, another for G, and another for R (Table 2). But you only used it to find the best fit. Now you'll use all entries to build PDFs. First, use these data to compute the denominator in eq. 30, and then use the same equation to compute the $P(p_i|D)$ probability associated to each model in the grid. Use a flat prior $P(p_i) = 1$, meaning that all models are, a priori, equally possible.

Now loop over your x_k grid. For each k, loop over your $i = 1 \ldots N_{library} = 27846$ table, adding up the probability of all models whose $\log t_i$ is within $x_k \pm \Delta x/2$. In the end you'll have a PDF(x_k) array. Notice that a given x_k-bin will contain models of wildly different Z's, A_V's and $M_\star$'s. We're marginalizing over them cause all we care about is the PDF for ages. We're collapsing a 4D space to 1D.

♣ *Plot your PDF($\log t$). Plot the results for B, G & R in a single frame. It is instructive to mark the best values of $\log t$ found in the previous exercise.*

♣ *Now that you've done it for $x = \log t/yr$, it's easy to do it for $x = \log Z/Z_\odot$, $x = A_V$ and $x = \log M_\star/M_\odot$, so do it.*

♣ *If you were to plot a PDF for, say, the M_g^{mod} values, what should it look like? While you're on it, why not do it and check if you guessed right? Tip: Although equivalent, it's more instructive to plot* *the PDF for* $\delta_g = (M_g^{mod} - M_g^{obs})/\sigma_g$.

Exercise 5.4 - Bayesian SED analysis - PDF summaries. *PDFs are cool (they represent the complete solution to an inference problem), but if you ask them "What is the value of* t*?" they'll answer you a function, not a number. It's therefore useful to summarize the PDF through statistics like its mode or its mean* ($\overline{x}$)*. Also, from its second moment* ($\overline{x^2}$) *you can derive* $\sigma(x) = \sqrt{\overline{x^2} - \overline{x}^2}$, *a standard estimate of the uncertainty in your estimate of* x*. If you prefer robust statistics, you can easily find the median* ($x_{50\%}$) *and the* x*-interval around the median which contains, say, 68% (equivalent to "1 sigma") of the probability, or 95% ("2 sigma"), etc.*

♣ *You're now equiped with numbers to refill Table 2, this time quoting values and uncertainties for the parameters. For each parameter, quote it's mean* $\pm$ *its standard deviation.*

Exercise 5.5 - Bayesian SED analysis - 2D PDFs. *It's simple to generalize the 1D-PDF exercise above to 2D. Pick 2 variables, like* $x = \log t$ *and* $y = A_V$*. Bin them in a uniform grid:* $x = x_k$, $y = y_l$, *with* $k = 0, 1 \ldots N_x - 1$ *and* $l = 0, 1 \ldots N_y - 1$*. You now have to compute a matrix* P_{kl} *containing the total probability in each* $(x_k \pm \Delta x/2\,,\, y_l \pm \Delta y/2)$ *2D bin (representing a box in an x-y diagram). The result will now be an image* P_{kl} *representing the joint PDF of* x *and* y.

♣ *Plot 2D PDFs for all pairwise combinations of* $\log t/yr$, $\log Z/Z_\odot$, A_V *and* $M_\star$*. Interpret your plots.*

Exercise 5.6 - Playing with priors. $P(p_i)$ *priors often disappear from the analysis (ie, they are set to a constant which cancels out in eq. 30). Let's illustrate how to incorporate more informative priors in the analysis.*

♣ *Our library allows for ages in the 0–20 Gyr range. If you believe cosmologists, the Universe has 14 Gyr, so models with* $t > 14$ *Gyr are absurd. You should thus re-do all you did by setting* $P(p_i) = 0$ *for all models where* $t_i > 14$ *Gyr (equivalent to eliminating them from the library).*

Suppose now that your religion postulates that stars in the Universe have a log-normal (i.e., a gaussian in the log) distribution of Z*'s centered on the solar value, such that*

$$P(p_i) \propto e^{-\frac{1}{2}\left((\log Z_i - \log Z_\odot)/\sigma_{\log Z}\right)^2}$$

This does not forbid library entries with $Z \neq Z_\odot$, but it gives them less weight. How much less? Much much less if $\sigma_{\log Z}$ is small (say, 0.1 dex), and not much less if $\sigma_{\log Z}$ is large (say, 1 dex).

♣ *Re-do your plots of 1D-PDFs for all parameters adopting $\sigma_{\log Z} = 0.1$, 1 and 10. The latter case ($\sigma_{\log Z} = 10$) is such a wide prior that in practice it differs little from no prior at all, so your results should be essentially the same you found before.*

♣ *You can play this game for other priors. For instance, you can now join the church-of-the-unpolluted-cosmos, which does not believe in dust ($A_V = 0$), or defy current cosmology and adopt an Universe 1 Gyr old (so only $t \leq 1$ Gyr is allowed). Etc, etc. These are stupid examples, of course, but they illustrate how to use prior-beliefs when you have them.*

Exercise 5.7 - Playing with errors. *Redo the PDF plots above changing the σ_b given in Table **??**. Make them both larger (eg., $\sigma_b \leftarrow 3\sigma_b$) and smaller (eg., $\sigma_b \leftarrow \sigma_b/3$). Do the results match your expectations?*

The library used in these exercises is, I've said it before, too simple, so don't be surprised if you found bad and/or crazy results. For starters, it's based on SSPs. This may be good for stars clusters (CF & González Delgado 2010) or massive ellipticals, but not for galaxies in general. The exercises above illustrate the procedure, but in real work one needs more realistic libraries. These can be built from models for composite stellar populations, like the exponential models described in §2. Even those are too simple, so in pratice people build libraries with more complex SFHs (eg, superimposing bursts on a smooth ψ function). In the Bayesian framework, the library makes part of your *prior*.

Exercise 5.8 - A less ridiculous ($\sim$ more realistic) library. *Let's reanalize B, G and R, this time using a comparison library composed of models where the SFH is that dictated by the exponentialy decaying SFRs of eq. 4. It's not smart to vary t_0 and τ separately (do you see why?), so let's parametrize the SFH in terms of t_0 and $\theta \equiv \tau/t_0$. The 2 other parameters in the grid will be A_V and Z. Use $\log t_0/yr = 6, 6.2, \ldots 10.2, 10.4$ and $\log \theta = -2, -1.8 \ldots 0.4, 0.6$. Use the 6 BC03-given values of Z and the same $A_V = 0, 0.1 \ldots 1.9, 2.0$ grid for the V-band extinction. Notice that this grid has one dimension more than the SSP-grid one, but $N_{library}$ is not so different (because of the choice of sampling, which you may wanna play with).*

From here on the procedure is just as before: For each model, generate M_b^{mod}'s for a $1M_\odot$ SFH and find the $M_\star$ which best scales up the predicted SED to the observed levels. Tabulate the model parameters $p_i = (t_{0,i}, \theta_i, Z_i, A_{V,i})$ along with the corresponding $M_{\star,i}$ (which can also be considered a parameter, but not a grided-one), the χ_i^2 and the model $M_{b,i}^{mod}$ for b = u, g, r, i, z. You may also want to tabulate quantities which are not explicit parameters of the model like τ_i, the mass weighted mean log age ($\langle \log t \rangle_{M,i}$ and its cousins), etc. These are all by-products of t_0 and θ, but you might want to obtain PDFs for them explicitly. With this big table, you can now compute the posterior probability $P(p_i|D)$ and then marginilize it to obtain 1 and/or 2D PDFs for things you find interesting. This will give you plenty plot-me-material for galaxies B, G and R.

There're many things you can re-do with this new library. To be specific:

♣ *Plot 1D PDFs for* $\log t_0$, $\log \theta$, Z, A_V *and* $M_\star$.

♣ *Compute the mean and std deviation of* $\log t_0$, $\log \theta$, Z, A_V *and* $M_\star$. *The latter 3 quantities were also estimated in two different ways before (Table 2), so compare these results.*

In the end, it's illustrative to compare things you found with one library to what you find with another one. Ultimately, this would be a comparison of results derived under different priors, since libraries are priors. Overplotting the PDFs of $\log t$ *(with the SSP-library) to the one for* $\log t_0$ *derived here, for instance, is a good exercise. The same applies for* Z, A_V *and* $M_\star$. *You might also want to enlarge the exponentially decaying SFHs library by using finer grids (more models), and/or adding the SSP models to the CSP ones. If the SSP models are realy bad approximations, they should weight very little and thus affect little-to-nothing the results, ...*

5.3 More and less data

All the above was based on ugriz photometry, a rather limited data set. In practice, sometimes you'll have even less data and sometimes you'll have more. Qualitatively, one expects that less data provides less stringent contraints in the parameters, reflected in broader PDFs, while more data should have the opposite effect. The exercises below give you the chance of checking this.

Exercise 5.9 - Less data: gri magnitudes. *Re-do the analysis above, this time considering only gri magnitudes. Specifically,*

♣ *Plot 1D PDFs for* $\log t_0$, $\log \theta$, Z, A_V *and* $M_\star$.

♣ *Compute the mean and std deviation of* $\log t_0$, $\log \theta$, Z, A_V *and* $M_\star$.

♣ *Do the results match your expectations?*

Exercise 5.10 - More data: ugriz + FUV, NUV & JHK magnitudes. *What if you were given data from the UV to the NIR? Re-do the analysis above, this time considering all the data in Table 1. R_λ filter transmissivities file available from our web dump.*

♣ *Plot 1D PDFs for $\log t_0$, $\log \theta$, Z, A_V and $M_\star$. Note: It's instructive to do these plots on top of those of the previous exercise.*

♣ *Compute the mean and std deviation of $\log t_0$, $\log \theta$, Z, A_V and $M_\star$.*

♣ *Do the results match your expectations?*

This kind of exercise is also useful to help you figure out which bands are critical to contrains this or that parameter, something which might help you design your survey or define sample selection criteria out of existing data. You can also turn the problem around, and predict measurements in a given band out of your analysis of data in other bands. Why woud you do that? At least for one reason: So far we have treated the evolutionary synthesis models as perfect, but this is just not so. Despite the progress, there are still phases of stellar evolution which are not fully comprehended. TP-AGB stars are a well known example. Since these stars affect mostly the NIR range, extrapolating results obtained from the UV-optical to the NIR, and doing so for libraries which employ different recipes for TP-AGB stars, one might be able to invert the whole logic of this course and use galaxies to learn about stars!

5.4 Even more data: ALHAMBRA & JPAS

Intermediate-narrow band photometric surveys are going to be in the headlines in the near future. The COMBO17 survey is one example. ALHAMBRA is another one, and JPAS will take it to an extreme. Presumably, the more data the better. Schoenell, in his MSc thesis, simulates JPAS data and uses an approach similar to the one above, but replacing model SEDs by observed ones, using libraries made up of actual SDSS galaxies, whose spectra were convolved with the 56 JPAS filters. This is why we spent so much time on photometry in a spectral analysis course.

Much of the focous of these surveys is on deriving photometric redshifts, a re-born industry in the past decade. But galaxies are more than their z, and these data are also useful to do things like you've just done.

5.5 The recent literature

Maaany articles in the literature do stuff similar to what you have just done. They were purposedly not mentioned above, not to distract you from our DIY philosophy.

Kauffmann et al (2003), Brinchmann et al (2004), Gallazzi et al (2005), Salim et al (2007), Dye (2008) are some examples I know. Walcher et al (2006, 2008) are also good examples, and quite didatic too. http://xxx.lanl.gov/pdf/1208.6419.pdf is an example of SED fitting of star clusters in M31. Etc, etc. You can find many references in Walcher et al (2010).

Exercise 5.11 - DIY literature search. *Find 3 papers published in the past 5 years which make use of the techniques we've studied.*

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6 Spectral Indices

Indices are quantities which are meant to summarize a spectrum. Colors like $g - r$, $FUV - NUV$ or $V - K$[4] trace, well, colors. As you've learned yourself (§1), colors do give you a rough idea of what kind of stellar populations you have (barring reddening effects, of course). As you've also seen in §1, absorption line strengths also give you the the same of information, with the advantage that, because of the small λ range, they are not affected by dust. The so called 4000 Å break, which looks like but is not an actual discontinuity, gives you a very useful tracer of the typical age of the stellar population.

Being a full spectral analysis person, I do not do much index-base work. Because of this, plus time-constraints and plain lazyness, we'll do only a very basic excursion through index-wok, just to give you an overall feeling of the thing. In the future this section should (hopefuly) grow to something more useful.

6.1 $D_n(4000)$ and $H\delta_A$

Bruzual (1983) defined the 4000 Å break index as can be defined in

$$D(4000) = \frac{\langle F^+ \rangle}{\langle F^+ \rangle} = \frac{(\lambda_2^- - \lambda_1^-)}{(\lambda_2^+ - \lambda_1^-)} \frac{\int_{\lambda_1^+}^{\lambda_2^+} F_\nu d\lambda}{\int_{\lambda_1^-}^{\lambda_2^+} F_\nu d\lambda} \tag{31}$$

which you can see is the ratio of mean F_ν fluxe in a $\lambda_1^+ \rightarrow \lambda_2^+$ red window (in the denominator) to the mean F_ν in $\lambda_1^- \rightarrow \lambda_2^-$ blue window. Bruzual used $(\lambda_1^-, \lambda_2^-) = (3750, 3950)$ for the blue window and $(\lambda_1^+, \lambda_2^+) = (4050, 4250)$ for the red one. Balogh et al (1999) proposes narrower windows:

[4]Note the convention of mag in smaller-λ minus mag in larger-λ, such that the color index gets numerically larger as the thing gets redder

$(\lambda_1^-, \lambda_2^-) = (3850, 3950)$ and $(\lambda_1^+, \lambda_2^+) = (4000, 4100)$. This latter choice has been adopted in many recent influential papers, most notably Kauffmann et al (2003). Because of the $_n$arrower widows, an $_n$ subscript has been appended to $D(4000)$:

$$D_n(4000) = \frac{(4100 - 4000)}{(3950 - 3850)} \frac{\int_{4000}^{4100} F_\nu d\lambda}{\int_{3850}^{3950} F_\nu d\lambda} = \frac{\int_{4000}^{4100} \lambda^2 F_\lambda d\lambda}{\int_{3850}^{3950} \lambda^2 F_\lambda d\lambda} \tag{32}$$

Notice that I've already changed from F_ν to F_λ.[5] Because of the narrow windows, few people worry about the effect of reddening on $D_n(4000)$, so we'll do the same.

Let's now define another index which become popular with SDSS papers: $H\delta_A$. As other balmer lines, an Hδ absorption line requires an electron in the $n = 2$ level, which only happens for temperatures of order 10000 K. Hotter stars have all all the H ionized, whereas cooler ones have their HI in the fundamental state. Since temperature is related to stellar mass (in the MS) and this to the age of an SSP, Balmer line strengths have a non-monototic weak $\rightarrow$ strong $\rightarrow$ weak again behaviour, which is what makes them interesting (and different from most other absorption lines).

The particular way of quantifying the strenght of a Hδ absorption we'll chose is that of Worthey & Ottaviani (1997). As usual in absorption line index definitions, one defines two side-bands (one to the blue and the other to the red of the feature), and a central "feature" band. For $H\delta_A$ the side bands are at $\lambda = 4083.50$–4122.25 and $\lambda = 4128.50$–4161.00, while the feature band is $\lambda = 4041.60$–4079.75 Å. Quoting Worthey & Ottaviani: *Absorption-feature indices are composed of measurements of relative flux in a central "feature" bandpass and two flanking "pseudocontinuum" bandpasses... Once the average fluxes in the pseudocontinuum bandpasses are found, a line is drawn between their midpoints to represent the "continuum" from which an index is measured by integration within the "feature" bandpass; the index is then expressed in terms of an equivalent width.*

To recap: Measure the mean F_λ in the flanking pseudo continuum bands and use them to define a straight line $C_\lambda = a\lambda + b$ continuum spectrum. Then compute $r_\lambda = (F_\lambda - C_\lambda)/C_\lambda$, and do $EW = \int r_\lambda d\lambda$ over the feature band to obtain the EW of the line. Use the windows defined above and you'll have $H\delta_A$.

It's time to see what $D_n(4000)$ and $H\delta_A$ are good for.

Exercise 6.1 - *Following the definitions above, compute $D_n(4000)$ and $H\delta_A$ for the BC03/Chab/Padova1994 SSP spectra.*

♣ *Plot $D_n(4000) \times \log t$. Use a different color for each of the 6 Z's.*

[5] To be honest, I have never figure out whether Balogh, Kauffmann nor all the papers which use $D_n(4000)$ do include these λ^2 terms (which come from $F_\nu = F_\lambda |d\lambda/d\nu|$), but this should make only a $\sim 4\%$ difference.

♣ *Plot* $H\delta_A \times \log t$. *Use a different color for each of the 6 Z's.*

♣ *Plot* $H\delta_A \times D_n(4000)$. *Use a different color for each of the 6 Z's. Overplot with filled circles the values for* $\log t = 6$, *7, 8, 9 and 10 to given you a visual guide of how an SSP evolves in this diagram.*

Exercise 6.2 - *Repeat the exercise above for the BC03/Chab/Padova2000 SSP spectra to check how things change with the choice of evolutionary tracks.*

Exercise 6.3 - *Repeat the exercise above for the models based on the Salpeter IMF. Do things change? By how much?*

6.2 $D_n(4000) \times H\delta_A$

The $D_n(4000) \times H\delta_A$ diagram played a central role in Kauffmann et a's analysis of SDSS galaxies. Soon after that paper, much more ellaborate methods (based on full specral synthesis techniques) to study stellar populations from the same data were published. Yet, the essence of what the data were telling us was already capture with what we now see as a very simplistic way of analysing spectra. The take-home-lesson is that it is well-worth to do a simple analysis! A more refined analysis may give you more refined results, but it'll hardly change the global picture in any significant sense.

How can one use $D_n(4000)$ and $H\delta_A$ to infer stellar population properties? Guess what: Just as you've done for photometric data in §5. In particular, with Bayesian techniques like those in §5.2. It's all really the same thing. Build your reference library, compute your chosen observables (magnitudes in §5 but $D_n(4000)$ and $H\delta_A$ now) for each model in your library, then compare your observed data to each model and build your PDFs. Just as you did!

Well, there's a minor not-exactly-like-it detail. Both $D_n(4000)$ and $H\delta_A$ are (in thermodynamics-lingo) "intensive" quantities. They cannot telll you if the galaxy has 1 gram or $10^{11} M_\odot$. But they can tell you what is the mass-to-light ratio of a system, in any waveband. Kauffamen et al (2003) estimated M/L_z from their Bayesian analysis of $D_n(4000)$ and $H\delta_A$, and then used the z-band photometry from the SDSS to derive stellar masses and other "extensive" quantities.

Measuring the $H\delta$ absorption requires removing it's nebular emission component, when present. This is a tricky business[6] we do not want to go into, so let's keep the exercises below in a theoretical

[6] One way to do it is to measure stronger Balmer lines, like $H\alpha$ and $H\beta$. Once you know the flux of these, you can predict the emssion flux from $H\delta$, including a reddening correction. This flux can then be discounted from your

world.

Exercise 6.4 - *Use the library of exponential SFHs built in §5. To recall, the parameters in that library were t_0, $\theta \equiv \tau/t_0$, A_V and Z. For each model, compute $D_n(4000)$ and $H\delta_A$. We now want to see how each of the library parameters (or things deduced from them, like mean ages or M/L ratios) translate into a location in the $D_n(4000) \times H\delta_A$ diagram.*

This is a simple thing to do, but difficult to visualize results. Here's one way to go about it. Call $x \equiv D_n(4000)$ and $y \equiv H\delta_A$ and pixelate the xy plane in a grid $x_k = x_0 + k \times \Delta x$ and $y_l = y_0 + l \times \Delta y$. Pick a generic parameter G; to fix ideas, consider $G = \log t_0$. For each (k,l) pixel you will have $m = 1 \dots N_{k,l}$ models in your library which produce x within $x_k \pm \Delta x/2$ and y within $y_l \pm \Delta y/2$—but notice that many of the (k,l) will be empty. Looping over the xy grid (as we did before in §5 to build 2D PDFs) you can easily fill in a 3D array $G_{k,l,m}$ which gives you the distribution of G-values ate each xy pixel. To produce something you can plot, compute the 1^{st} and 2^{nd} moments of G, which will give you 2D-maps of $\overline{G}$ and its std deviation $\sigma_G = (\overline{G^2} - \overline{G}^2)^{1/2}$ as a function of x and y. You can use robust statistics (median and some interpercentile range) if you prefer.

Once you've setup your scripts, do these images for different G's. The plots should be read like this: If σ_G is generally large (compare to the values of G) throughout the $D_n(4000)$-$H\delta_A$ diagram, than G is not well constrained by these two indices, and vice versa. You should find both cases, ie, G's which are well constrained by a $(D_n(4000), H\delta_A)$ pair, and G's which aren't.

Note: What you just did is very similar to Kauffmann et al (2003). Look up her figures.

Exercise 6.5 - *Plot Bayesian 1D-PDFs for $G = \log t_0$, $\log \tau$, Z, $\log M/L_{5635}$ and A_V (always marking the mean, the $\pm 1\sigma$, the median and the 16–84% percentile interval) for the following hypothetical data sets:*

♣ $D_n(4000) = 1.3 \pm 0.1$ *and* $H\delta_A = (4 \pm 1)$Å.

♣ $D_n(4000) = 1.3 \pm 0.1$ *and* $H\delta_A = (8 \pm 1)$Å.

♣ $D_n(4000) = 2.0 \pm 0.1$ *and* $H\delta_A = (-1 \pm 1)$Å.

Obs: I haven't done it, but I anticipate some results will be strange at first sight…

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measured $H\delta_A$.

6.3 The synthesic domain

Pelat (1997) introduced several important concepts (little used or acknowledged in the litetature though) useful in the analysis of galaxy data. One of them is the *Synthetic Domain*. He was thinking about fitting the equivalent widths (W_λ) of absorption lines, but the concept is more general. You can, for instance, apply it to any color-color diagram, or to the $D_n(4000)$-$H\delta_A$ diagram just discussed.

Suppose you want to describe a composite stellar systems in terms of convex combinations (ie, linear combinations where the coefficients are ≥ 0 and add up to 1) of base elements, like SSPs. (This is just a complicated way of stating the obvious fact that a composite stellar population is a sum of SSPs, as used in eq. 3, for instance.) Say you have $j = 1 \ldots N_\star$ base elements. Then pretend that only 2 of these base elements are present, and draw the corresponding mixture lines in you index-index diagram. To do this, compute you indices supposing a fraction x of the light at some reference wavelength comes from component j, so that $1 - x$ comes from component k. Looping from $x = 0$ to 1 gives you the mixture line in the I_A-I_B index-index diagram. Do this for all $N_\star(N_\star - 1)$ pair-wise combinations and the result will be a messy diagram!

Messy, but useful. First, its outer contour gives you the synthetic domain, region in the I_A-I_B

The you can do .... Suppose you have N such elements. Their W_λ's are $W_{\lambda,j}$, with $j = 1 \ldots N$. Suppose you are now interested in ...the combination of all pairs of base elements...

7 Full spectral fitting

spectral algebra .. convert $\vec{x}(\lambda_1)$ to $\vec{x}(\lambda_2)$

light ($\vec{x}$) to mass (($\vec{\mu^{ini}}$ and $\mu^{\vec{cor}}$) conversion

progapage uncertainty in light to mass... (use covariances?)

7.1 STARLIGHT

My STARLIGHT code (CF et al 2004, 2005, Mateus et al 2006; see manual @ www.starlight.ufs.br) combines spectra from a base (usually SSPs, but it can be anything!) to produce a model spectrum M_λ with best matches an observed spectrum O_λ. Ideally, the user should also supply an error spectrum (ϵ_λ) and a bad-pixel-flag b_λ (an integer which, when ≥ 2 informs that a pixel is bad by whatever reason). Also, a mask spectrum m_λ tells the code to ignore data in pre-defined spectral windows. This mask is normally used to inform emission line windows, but it can be used to mask-away other things, like regions where you know your base models are problematic, or faulty regions not accounted for in the b_λ bad-pixel-flag.

STARLIGHT inherits many of the ideas and conventions first put forward by Eduardo Bica (Bica 1988 and tons of following papers). In particular, it follows the convention of expressing the strength of a given base component by its fractional contribution to the total light at a chosen reference/normalization λ. This is a source of confusion, because what people are usually after (at least nowadays) is the (fractional or absolute) mass associated to a given component. It is undertandable that people seek to find masses, but it's very important to understand/realize that one sees light, not mass. This trivial fact-of-life must not be overlooked. Because L and M relate in a highly non-linear way ($L \propto M^\alpha$ in stars, with α of order 3!), a tiny amount of light may end up translating to a lot of mass, and vice versa.

Exercise 7.1 - *Using the Padova1994/chab/m62 models, first determine $l_\lambda(t)$ for $\lambda = 5635$ Å.*
$\boxed{\mathbf{!\odot j\odot!}}$ *Use the best-SSP-fit feature and a sample of LRGs to find an age-redshift relation...* ,,

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$\boxed{\mathbf{!\odot j\odot!}}$ some of the exercises/challenges I gave them...

8 A taste of emission line analysis

This course is not about emission lines, but you should least have a rought idea of how to deal with them and what they can tell you. This section aims to fill this gap through a series of highly simplified hands-on exercises.

The file $\boxed{\mathbf{!\odot j\odot!}}$ contains STARLIGHT output files for 10 galaxies, whose distances and stellar masses are listed in $\boxed{\mathbf{!\odot j\odot!}}$. Your task is to measure Hβ, [OIII]λ5007, Hα, and [NII]λ6584, and use them to derive a series of physical properties and diagnostics.

Exercise 8.1 - *We 1st need to measure luminosities (L, in $L_\odot$), the underlying continua (C_λ, in $L_\odot$/Å) and equivalent widths (W, in Å) for four emission lines: Hβ, [OIII]λ5007, Hα, and [NII]λ6584. Here's a script of how to do this:*

1. *Re-scale the observed (O_λ) and model (M_λ) spectra to units of $L_\odot$/Å. To do this, first multiply them by the normalization flux fobs_norm (given in units of 10^{-17} ergs s cm^{-2} Å^{-1}) and then by $4\pi d^2$, such that the end spectra are in units of $L_\odot$/Å.*

2. *Compute the residual spectrum $R_\lambda = O_\lambda - M_\lambda$.*

3. *Measure emission line luminosities (L, in $L_\odot$) integrating $R_\lambda d\lambda$ in adequate windows. If the residual is systematically negative or positive in the neighbourhood of a line you may need to account for this with a local "residual continuum" to get a precise measurment. Alternatively, you may want to fit a gaussian to each line (instead of integrating $R_\lambda d\lambda$) and using its amplitude and width to compute the integrated luminosity.*

4. *Measure the continuum C_λ at the central wavelength of each of the emission lines. This is usually done defining narrow (a few tens of Å) bands on the blue and red sides of the line, computing*

	luminosity $[L_\odot]$				continuum $[L_\odot/\text{Å}]$				eq. width $= L/C_\lambda$ [Å]			
Galaxy	Hβ	[OIII]	Hα	[NII]	Hβ	[OIII]	Hα	[NII]	Hβ	[OIII]	Hα	[NII]
...												
...												

the mean flux in each and interpolating linearly ($C_\lambda = a\lambda + b$) between them to get the value of C_λ at the central wavelength you want.

Tip: You may prefer to measure C_λ using the fitted (M_λ) rather than the observed (O_λ) spectrum, since M_λ is essentially problem free.

5. *The equivalent width (W) may be defined as the line luminosity divide by its continuum. (This is not exactly the textbook definition, but it should not matter.)*

♣ *List your L's, W's and C's in a "data" table like 1. Tables like these (though not in these units) were common in the 80's and 90's, before the numer of lines grew to $>$ or $\gg 100$.*

In serious work you should also estimate the uncertainties in all these stuff. We'll ignore errors in this section, thoughg you'll likely see whether they matter...

Exercise 8.2 - Nebular extinction. *Recombination theory tells us that (except in exceptional circunstances) the ratio of Balmer lines is nearly independent of the detailed physical conditions (eg, density and temperature) of the ionized gas. The Hα/Hβ ratio (sometimes called the "Balmer decrement"), in particular, should be 2.86.*[7] *Look up you data table and you'll see that this is generally not the case! Barring gross errors, the reason is dust. If the nebula is seen through a screen of dust, the observed luminosities are smaller than the intrinsic ones by*

$$L_\lambda^{obs} = L_\lambda^{int} \times 10^{-0.4A_\lambda},$$

and since $A_\lambda = A_V q_\lambda$ increases towards the blue, the observed $H\alpha/H\beta$ ratio should be larger than $(H\alpha/H\beta)_{int} = 2.86$. Turn this around to show that

$$A_V^{Neb} = \frac{2.5}{q_{H\beta} - q_{H\alpha}} \log \frac{(H\alpha/H\beta)_{obs}}{(H\alpha/H\beta)_{int}}$$

[7] This applies to nebulae excited by young stars. A slightly larger value (abot 3 or 3.1) applies if the ionizing source is harder (like in an AGN) but we'll overlook this "detail" (and others!) here.

This nebular extinction is the first property derived from your measurements. Naturally, you'll need to adopt a q_λ law. Use the CCM law in this exercise. Sometimes you'll obtain $A_V < 0 \dots$

♣ *Once you have A_V^{Neb}, you can (and should) correct all the line luminosities. You can also do so for the underlying contiinum luminosities, but that entails one further assumption, namely, that L's and C_λ's are seen through the same column of dust. Does it sounds a reasonable thing to assume?*

♣ *What values of A_V^{Neb} would change an observed Hα/Hβ ratio by 0.1, 0.2 and 0.3 dex?*

♣ *What values of A_V^{Neb} would change an observed [OIII]/[NII] ratio by 0.1, 0.2 and 0.3 dex?*

♣ *What values of A_V^{Neb} would change an observed [NII]/Hα ratio by 0.1, 0.2 and 0.3 dex?*

Exercise 8.3 - SFR(Hα). *In §3 we've seen how to estimate the SFR in the past ~ 10 Myr using the (extinction-corrected) Hα luminosity, so do this for our 10 galaxies.*

Exercise 8.4 - Nebular metallicity: O/H. *There are tons of ways to estimate the nebular metalicity (more precisely, oxygen abundances) from emission lines, and tons of caveats in this business. To cut a long story short, let's use just two illustrative "strong-line-methods", based on the "N2" and "O3N2" indices, defined as*

$$N2 \equiv \log \frac{L^{int}_{[NII]}}{L^{int}_{H\alpha}}$$

$$O3N2 \equiv \log \frac{L^{int}_{[OIII]}}{L^{int}_{[NII]}}$$

In both cases we correct for reddening (thats what the ^{int} superscript means), but in practice the correction ought to be neglibgible for $N2$. Oxygen abundances are usually quoted as $12 + \log O/H$ instead of O/H or $(O/H)/(O/H)_\odot$ ... (why make it simple if we're astronomers?). Pettini & Pagel (2004) propose the following calibrations

$$12 + \log O/H = 8.90 + 0.57 \times N2$$

$$12 + \log O/H = 8.73 - 0.32 \times O3N2$$

Regarding the latter, Staśinska (2006) prefers

$$12 + \log O/H = 8.55 - 0.25 \times O3N2$$

There are many others (see Kewley & Ellison 2008, ApJ, 681, 1183) but these 3 will do it for now. Apply all 3 to estimate the nebular metallicity of our 10 galaxies. Write the results both in terms of $12 + \log O/H$ *and units of* $(O/H)_\odot = 4.9 \times 10^{-4}$.

Exercise 8.5 - Diagnostics diagrams. *One of the major uses of emission lines nowadays is to classify a galaxy as SF, Seyfert, LINERS, composite, … Classification is a typical obsession of astronomers—sadly, an often overrated one. In any case, it's good to know whether our emission lines are SF or AGN like. If a galaxy has an AGN affecting its optical emission lines, the estimates of SFR and O/H above are wrong! It's thus good to know whether you've done such a mistake, and we'll use dignostic diagrams to check this.*

1. *Plot the galaxies in a BPT diagram: log [OIII]*$\lambda5007/H\beta$ *versus log [NII]*$\lambda6584/H\alpha$ *(Baldwin, Phillips & Terlevich 1981).*
2. *Plot the galaxies in a WHAN diagram:* $W_{H\alpha}$ *versus log [NII]*$\lambda6584/H\alpha$ *(CF et al 2010, 2011).*

♣ *Check which galaxies are not bona-fide SF galaxies. Do so using the Kauffmann et al (2003) and the Stasinska et al (2006) classification schemes for the BPT diagram, and the CF et al (2011) for the WHAN diagram. Note that for each galaxy you'll have 3 different classifications. (I can advance that they'll not be completely consistent…)*

Exercise 8.6 - Putting all together. *Wrap up the results of the above exercises in a table containing* $A_V[Neb$*, SFR,* $12 + \log O/H$ *(cf the 3 recipes outlined above), and the emission line classification (cf. the Kauffmann, Stasinska & CF classification schemes).*

♣ *Plot* $12 + \log O/H$ *versus the stellar mass* $M_\star$ *(given for free in this section). Mark AGNs (whose O/H estimates are surely wrong) with a different symbol. Overplot the mass-metallicity relation derived in the famous Tremonti et al (2004) paper. What do you conclude?*

You've measured emission lines and used them to perform some basic diagnostics. You've also done silly things on the way, just as we do in day-to-day spectral analysis. There's of course much much more to emission lines, but this is enough for our purposes.

9 Epiloge

10 References:

http://iopscience.iop.org/1538-3881/125/5/2348/fulltext/202203.text.html
http://www.roe.ac.uk/ifa/postgrad/pedagogy/2008_phillips.pdf
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