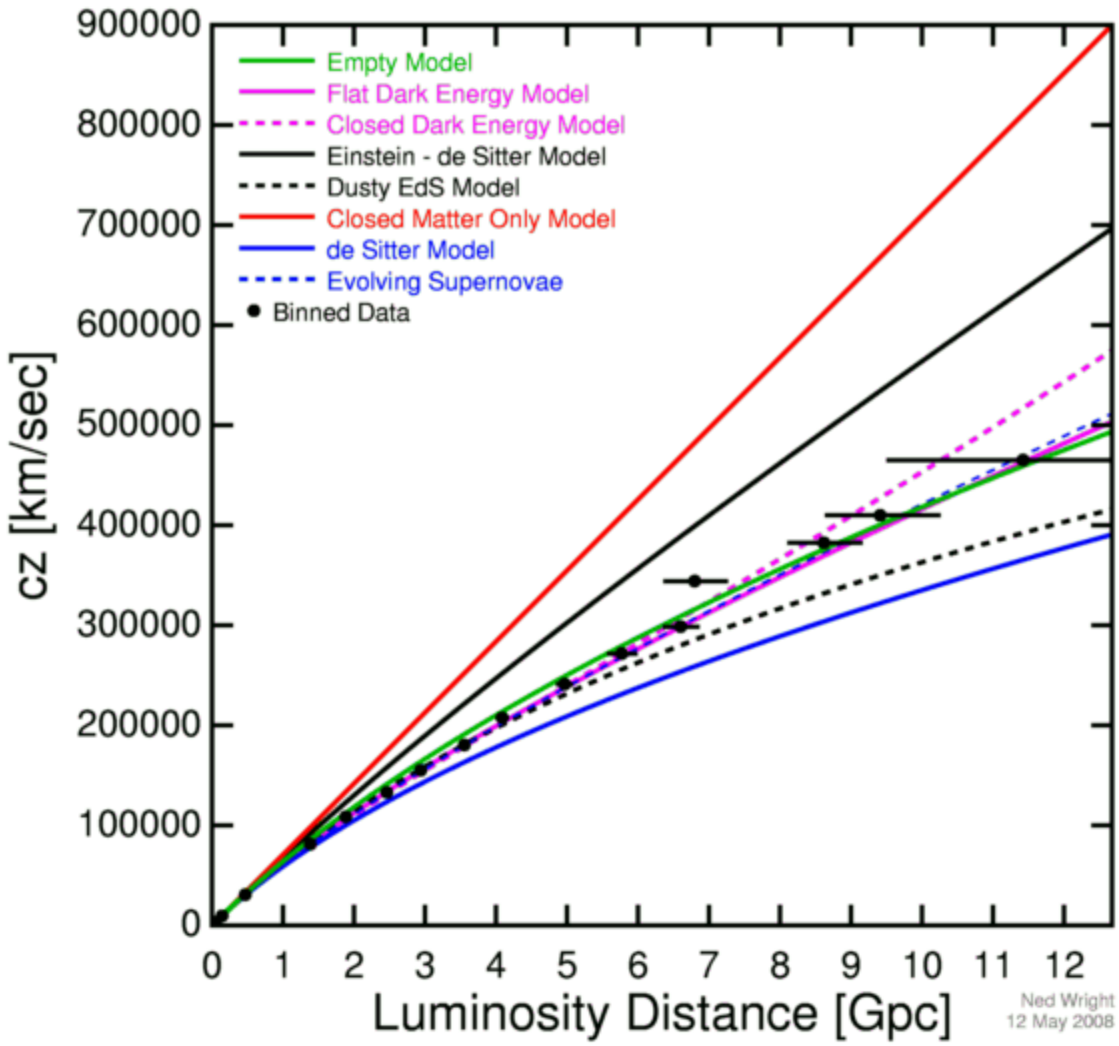
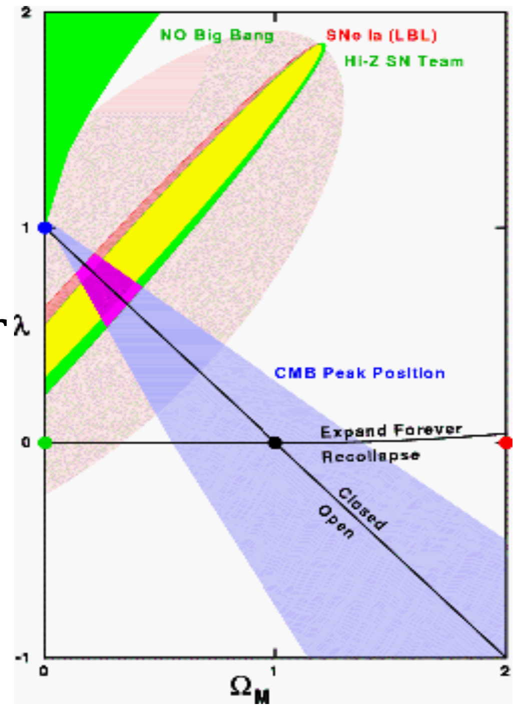
Measuring the Curvature of the Universe by Measuring the Curvature of the Hubble Diagram

Several groups are measuring distant supernovae with the goal of determining whether the Universe is open or closed by measuring the curvature in the Hubble diagram. The figure below shows a binned version of the latest dataset: Kowalski *et al.* (2008).

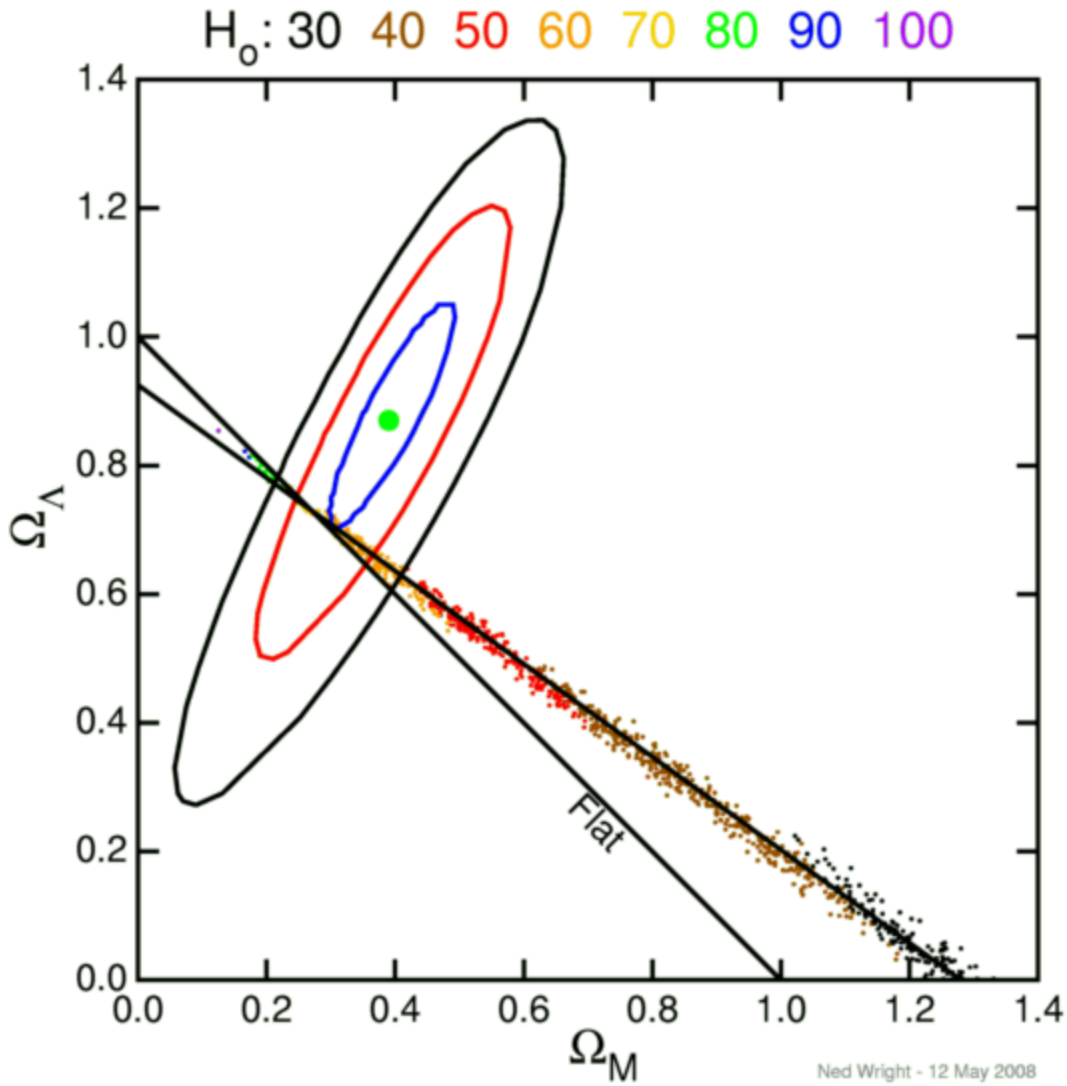


The curves show a closed Universe ($\Omega = 2$) in red, the critical density Universe ($\Omega = 1$) in black, the empty Universe ($\Omega = 0$) in green, the steady state model in blue, and the WMAP based concordance model with $\Omega_M = 0.27$ and $\Omega_V = 0.73$ in purple. This model gives $H_o = 71$ km/sec/Mpc which has been used to scale the luminosity distances in the plot. The data show an accelerating Universe at low to moderate redshifts but a decelerating Universe at higher redshifts, consistent with a model having both a cosmological constant and a significant amount of dark matter. The dashed black curve shows an Einstein-de Sitter model with a constant co-moving dust density which can be ruled out. The dashed purple curve shows a closed ΛCDM model which is a good fit to the data. The dashed blue curve shows an evolving supernova model which is also a good fit. Note that [power law a(t)](#) models where the scale factor is a power of the cosmic time can be ruled out.



Both the Supernova Cosmology Project and the High-z Supernova Team groups were the subject of news articles in Science, on 30 Jan 1998 and 27 Feb 1998. I have combined their two error ellipses along with another constraint from the circa 1998 knowledge of the location of the acoustic peak in the angular power spectrum of the CMB anisotropy. The two SNe groups gave very similar error ellipses, and the combined CMB-SNe fit indicates that a flat Universe with a [cosmological constant](#) is preferred. But the systematic errors on the SNe data, shown as the large grey (or pink) ellipse, could allow for a vanishing cosmological constant lambda. The red, black, green and blue circles on the Figure to the right are keyed to the colors of the curves on the

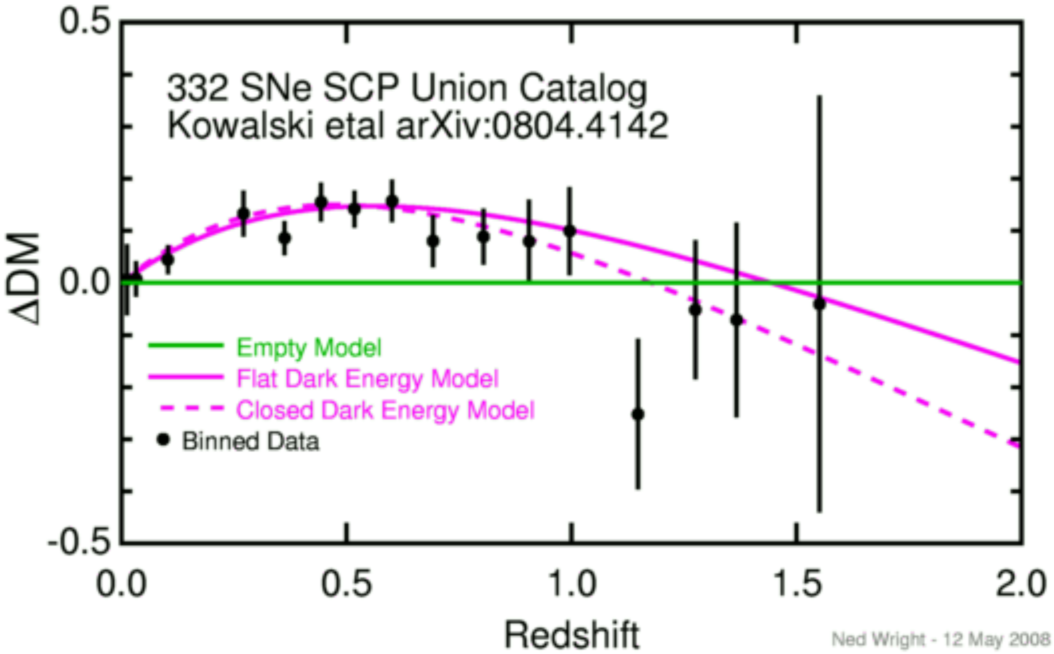
Figure shown above. A larger GIF file or a Postscript version of this figure are available.



The supernova data as of April 2008 published by Kowalski *et al.* (2008) provide the best fit, 1, 2 and 3 standard deviation contours shown as the green, blue, red and black ellipses in the figure at left. The CMB data using WMAP five year results provide the cloud of dots from a Monte Carlo Markov chain sampling of the likelihood function. The CMB degeneracy track does not follow the flat Universe line, but crosses the flat line at a point reasonably consistent with the supernova fit. Each CMB model has an implied Hubble constant which provides the color code for the dots. A model that fits both the supernova data and the CMB data has a Hubble constant that agrees reasonably well with the Hubble Space Telescope Key Project value of the Hubble constant.

The addition of high redshift supernovae has had two effects on the supernova error ellipse. The long axis of the ellipse has gotten shorter, and the slope of the ellipse has gotten higher. The best fit model has gotten closer to the CMB degeneracy track in absolute terms, and it has also gotten closer in terms of standard deviations in the Kowalski *et al.* (2008) dataset.

In the last few years distant supernovae with redshifts up to 1.755 have been observed by the Hubble Space Telescope. These objects show that the trend toward fainter supernovae seen at moderate redshifts has reversed. This reversal means that one possible alternative to the accelerating Universe as the explanation of the fainter supernovae at z near 0.5 can be rejected. This rejected alternative proposed that dust between galaxies made the distant supernovae fainter by absorbing some of their light. In the plot below, the brightness or faintness of distant supernovae relative to the empty Universe model is plotted *vs* redshift.



The green curve is the Ω=0 Universe. The solid magenta curve shows the best fit flat accelerating vacuum-dominated model. The dashed magenta curve is the best closed dark energy dominated fit to the supernova data alone.

The data points on the above plot come from my binning [Kowalski *et al.* (2008)](), which gives these normal points:

<z>	<ΔDM>	σ
0.01165	0.0060	0.0678
0.03231	0.0074	0.0342
0.10263	0.0445	0.0281
0.27094	0.1330	0.0441
0.36235	0.0859	0.0326
0.44353	0.1551	0.0372
0.51734	0.1418	0.0356
0.60119	0.1570	0.0408

0.69209	0.0804	0.0499
0.80419	0.0885	0.0535
0.90584	0.0796	0.0804
0.99577	0.0995	0.0845
1.14750	-0.2520	0.1446
1.27500	-0.0517	0.1333
1.36667	-0.0710	0.1869
1.55100	-0.0407	0.4000

My binning of the Riess *et al. (2007)* data table gives these binned normal points:

n	z_{min}	z_{max}	<z>	d(DM)	sigma
31	0.00700	0.02100	0.01484	-0.0464	0.1383
31	0.02300	0.05000	0.03352	0.0063	0.0691
16	0.05100	0.12400	0.07131	0.0725	0.0644
7	0.16000	0.24900	0.20671	0.0916	0.0878
18	0.26300	0.35900	0.32239	0.0751	0.0506
31	0.36900	0.46000	0.42323	0.1665	0.0406
31	0.46100	0.52600	0.49016	0.2700	0.0395
29	0.52600	0.62000	0.56921	0.1521	0.0375
20	0.62700	0.72100	0.67190	0.0969	0.0478
24	0.73000	0.83000	0.79029	0.0799	0.0519
17	0.83200	0.93000	0.87647	0.0464	0.0697
19	0.93500	1.02000	0.97011	0.0155	0.0696
4	1.05600	1.14000	1.11400	0.0168	0.1179
5	1.19000	1.26500	1.22280	-0.0870	0.1275
6	1.30000	1.39000	1.33533	-0.1505	0.0998
1	1.40000	1.40000	1.40000	0.0371	0.8100
1	1.55100	1.55100	1.55100	-0.4897	0.3201
1	1.75500	1.75500	1.75500	-0.5993	0.3501

where d(DM) is the difference between the distance modulus determined from the flux and the distance modulus computed from the redshift in the empty Universe model, and sigma is the standard deviation of the d(DM) in the bin. I use a robust statistical technique to get the binned values and therefore include both the Gold and Silver samples. I also include the low redshift supernovae which of course only affect the low z bin. But I have assumed a 1500 km/sec uncertainty in the redshift when computing the d(DM) which de-weights the low redshift bin.

I don't see much difference between the Gold+Silver data and the data restricted to Gold, but here is a binning of the Gold data alone:

n	z_{min}	z_{max}	<z>	d(DM)	sigma
22	0.01000	0.02100	0.01536	0.0186	0.1599
22	0.02300	0.04000	0.02986	0.0441	0.0892
18	0.04300	0.12400	0.06467	0.0387	0.0635
4	0.17200	0.26300	0.21600	0.1356	0.0912
12	0.27800	0.37100	0.33167	0.0720	0.0551
22	0.38000	0.47000	0.43777	0.1798	0.0446
22	0.47000	0.54000	0.50223	0.2119	0.0462
22	0.54300	0.64000	0.59268	0.1092	0.0417
11	0.64300	0.74000	0.69855	0.0930	0.0607
18	0.75600	0.85400	0.81217	0.0422	0.0607
13	0.86000	0.95400	0.91862	0.0140	0.0767
8	0.96100	1.05600	0.99863	0.1141	0.0912
4	1.12000	1.19900	1.14975	-0.0473	0.1273
4	1.23000	1.30500	1.26625	0.0566	0.1138
3	1.34000	1.39000	1.36667	-0.1848	0.1360
1	1.75500	1.75500	1.75500	-0.5993	0.3501

I have also thrown the ESSENCE dataset into the Riess *et al.* (2007) dataset, getting the followed binned dataset. I needed to add 0.022 mag from the μ values in Table 9 of Wood-Vasey *et al.* (2007) to make the sample of objects in common consistent with the Riess *et al.* scale.

n	z_{min}	z_{max}	<z>	d(DM)	sigma
37	0.00700	0.02400	0.01589	-0.0518	0.1180
37	0.02450	0.05800	0.03757	0.0040	0.0573
12	0.06100	0.16000	0.09475	0.1026	0.0752
14	0.17200	0.26800	0.22071	0.1097	0.0625
36	0.27400	0.37100	0.32989	0.0963	0.0374
37	0.37400	0.45500	0.42224	0.1698	0.0373
37	0.45900	0.51100	0.48408	0.2449	0.0371
37	0.51400	0.61000	0.55297	0.1687	0.0340
31	0.61200	0.71000	0.65503	0.0999	0.0358
21	0.71900	0.81800	0.77471	0.0535	0.0526
20	0.82200	0.91000	0.85905	0.0546	0.0644
21	0.92700	1.02000	0.96614	0.0469	0.0677
4	1.05600	1.14000	1.11400	0.0168	0.1179
5	1.19000	1.26500	1.22280	-0.0870	0.1275
6	1.30000	1.39000	1.33533	-0.1505	0.0998
1	1.40000	1.40000	1.40000	0.0371	0.8100

1	1.55100	1.55100	1.55100	-0.4897	0.3201
1	1.75500	1.75500	1.75500	-0.5993	0.3501

The table above is Table 1 from Wright (2007).

The table below is the Riess et al Gold plus the ESSENCE supernovae, from Table 2 in Wright (2007):

n	z_{min}	z_{max}	<z>	d(DM)	sigma
29	0.01000	0.02500	0.01694	-0.0351	0.1250
29	0.02500	0.05300	0.03612	0.0240	0.0667
10	0.05600	0.12400	0.07760	0.0742	0.0798
10	0.15900	0.24900	0.20320	0.1229	0.0735
27	0.26300	0.36300	0.31963	0.1054	0.0425
29	0.36800	0.45000	0.41490	0.1437	0.0390
29	0.45500	0.50800	0.48083	0.2032	0.0401
29	0.51000	0.60400	0.55145	0.1386	0.0389
25	0.61000	0.70700	0.64748	0.1190	0.0381
18	0.73000	0.83000	0.78883	0.0581	0.0587
10	0.83200	0.90500	0.86660	0.0073	0.0811
14	0.93500	1.02000	0.96957	0.0347	0.0760
4	1.05600	1.14000	1.11400	0.0168	0.1179
3	1.19900	1.23000	1.21967	0.0806	0.1434
5	1.30000	1.39000	1.34100	-0.1629	0.1054
1	1.75500	1.75500	1.75500	-0.5993	0.3501

Note that this Riess etal (2007) dataset is a compilation of data from many sources and there are indications that there are systematic differences between these subsets.

Observationally

$$d(DM) = 5 \log (H_o \,\mathrm{sqrt}[L/(4\pi F)]/[cz(1+z/2)])$$

while theoretically

$$d(DM) = 5 \log[Z(z)\, J([1-\Omega_{tot}]Z(z)^2)\, (1+z)/(z(1+z/2))]$$

with Z(z) and J(x) defined here. The Hubble constant used in computing the empty Universe

Milne model which is subtracted off is 63.8 km/sec/Mpc, to be consistent with Riess *et al.* (2007). Note that any fit to this dataset should include as a free parameter an adjustment to this Hubble constant, which gives a constant term in d(DM).

I found the following chi^2 values for fits to both the unbinned and the binned Riess *et al.* (2007) Gold+Silver data:

Name	Omega_m	Omega_vac	unbinned chi^2/df	binned chi^2/df
Best fit	0.55	1.15	290.4/289	12.1/15
Best flat	0.36	0.64	297.7/290	20.1/16
WMAP model	0.27	0.73	302.6/291	25.3/17
Milne	0.0	0.0	321.2/291	44.5/17
EdS	1.0	0.0	386.3/291	108.6/17
Evolving	1.0	0.0	295.8/290	18.2/16

The evolving model is the model with supernova luminosity evolving as a exponential function of cosmic time, which I discussed in astro-ph/0201196. This model is still a better fit than the flat vacuum-dominated model, but not to a significant degree.

I have also binned the γ-ray burst (GRB) data from Schaefer (2006):

n	z_{min}	z_{max}	<z>	d(DM)	sigma
1	0.17000	0.17000	0.17000	0.4532	0.3813
1	0.25000	0.25000	0.25000	0.4471	1.1402
2	0.43000	0.45000	0.44000	0.2069	0.3973
6	0.61000	0.71000	0.68000	0.4517	0.2367
7	0.78000	0.86000	0.82857	0.3608	0.2116
6	0.96000	1.10000	1.02000	-0.1046	0.2142
8	1.24000	1.51000	1.37625	-0.0509	0.1866

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5 1.52000 1.71000 1.60200  -0.2954  0.2299
8 1.98000 2.35000 2.17375  -0.0616  0.2020
7 2.44000 2.90000 2.65857  -0.5738  0.2612
8 3.08000 3.53000 3.30000  -0.4595  0.2219
7 3.79000 4.50000 4.10429  -0.8771  0.2301
1 4.90000 4.90000 4.90000  -0.5275  0.9500
2 6.29000 6.60000 6.44500  -1.1004  0.4628
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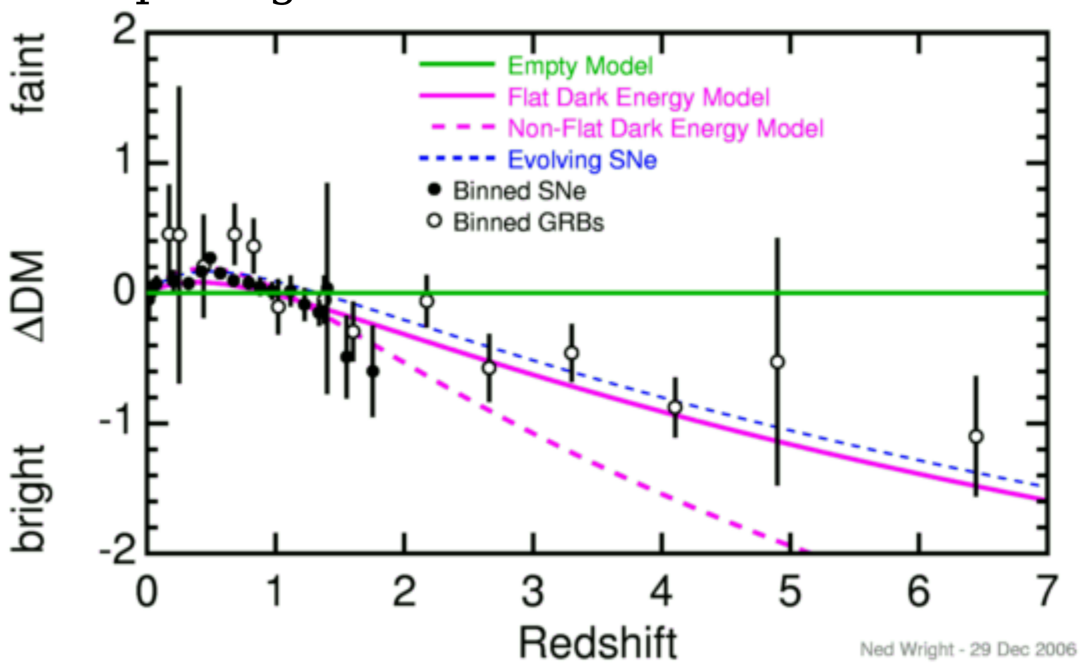
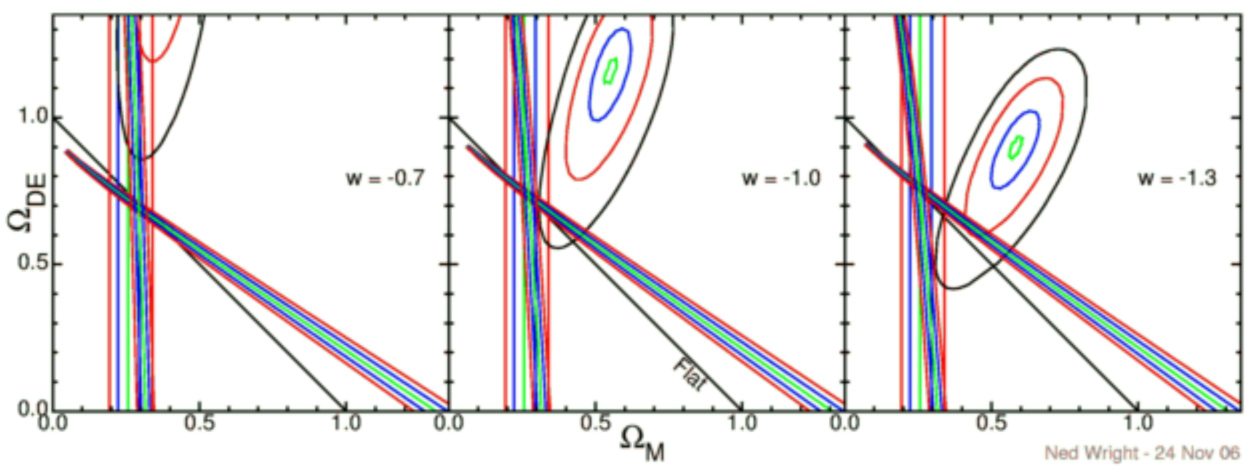
Note that Schaefer uses a different H_o than Riess but I have used the appropriate H_o (72 km/sec/Mpc) when computing the Milne model for this dataset.



The plot above shows the difference in distance modulus between the empty model and the supernova and the GRB binned data. It looks a bit inconsistent at redshifts near 0.5 but the residuals from the fits are not much bigger than the stated errors. When fitting to both the SNe and GRB datasets, there should be two free parameters for Hubble constant changes, one for each dataset. These free parameters can be thought of as adjustments to the overall luminosity calibration of

SNe and GRBs respectively.

With multiple datasets it is now possible to say something about the equation of state parameter w even without assuming the Universe is flat. The figure below shows the constraints from the Hubble constant (vertical lines), the baryon acoustic oscillations (nearly vertical lines), the CMB (tilted fan of lines), and the supernovae (ellipses). In each case green is right on (or 0.3 sigma for the supernovae), blue is 1 sigma, red is 2 sigma, and black is 3 sigma. H_o taken to be 71 +/- 5 km/sec/Mpc based on an average of the HST Key project, the SZ effect, the Cepheids in the nuclear maser ring galaxy NGC 4258, and the double-lined eclipsing binary in M33.



The three panels above show three different values of the equation of state parameter, w = -0.7, -1, and -1.3. Clearly if one assumes the Universe is flat the supernovae favor w = -1.3 which leads to a "Big

Rip". But if one looks only at the concordance between the four datasets, the standard flat ΛCDM model with w = -1 is preferred.

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